**Ten-Fold Tiling**

*a family of forms*

Unlike triangles, squares and hexagons, regular pentagons cannot be arranged to fill a flat surface without leaving gaps. As the art of geometric patterns developed in the Islamic world, artisans inevitably turned their attention to this challenge and discovered ingenious ways of creating designs using five- and ten-fold symmetries.

The diagram below shows a pattern unfolding from a subgrid of repeating regular decagons, placed edge to edge to leave curious bowtie-shaped hexagons. Stars are traced from the midpoints of the decagons’ edges forming pentagons at the decagons’ corners. The lines of the stars extend into the spaces between decagons to complete the pattern. This pattern is known in Persian as *Umm al-Girih*, the mother of patterns (knots), and its component shapes are the first generation in a whole family of forms (see page 54).

The constructions shown opposite are based on an Iranian method. Radial lines marking every eighteen degrees (*dotted*) are intersected by additional lines (*solid*) to give proportioning circles. These circles intersect the radial lines in delicate webs that give the vertices of the final pattern. The arabesque motifs used opposite are in the style of Mamluk Quran illumination.
Pentagrammaton
a second ten-fold family

Replace every regular pentagon in the Unm al-Girih, including the two overlapping pentagons of the large ten-pointed stars, with five-pointed stars to generate the fundamental pattern of a second ten-fold set of shapes. As with the Unm al-Girih this pattern's components are the first generation in a whole family of forms, some of which are shown opposite.

Both of the ten-fold shape-sets can be used to make a countless variety of patterns. For example, the wooden window shutters in the great Ottoman mosques of Istanbul bear a multitude of ten-fold designs, in some buildings seemingly without repetition. Two designs from the Sokullu Mehmet Pasha mosque are illustrated opposite with an example of the symbolism sometimes concealed in the number of a pattern's pieces.

Five-fold and ten-fold geometry embody the elegant golden section, the proportion formed when a line is cut such that the shorter section is to the longer as the longer is to the whole line (approximately 1.618). In the pattern below each distance that can be measured between corners or intersections on a line forms a golden section with the next smallest or next largest distance.

Arabic letters all have numerical values. This system, known as 'abjad,' was originally used for writing numbers before the decimal numerals were adopted. Nowadays it is used for its symbolic value. The pattern above right is composed of 165 pieces, the abjad total for 'La ilaha illa Allah.'—"There is no divinity but God." -- the quintessential statement of Divine Unity in Islam. A related example is shown in the frontispiece of this book; 99 pieces correspond to the traditional number of Divine Names.
Decimal Connections

between the two families

The basic ten-fold patterns on the previous pages can also be generated from a subgrid of regular decagons, pentagons and pentagon-based hexagons (below left). Placing pentagons and pentagram stars in this subgrid gives the two patterns (below right).

Arranging the two patterns this way suggests the possibility of varying the angles formed where paths cross at the subgrid’s edge midpoints. The *Umm al-Giriḥ* has angles of 108° at these fixed points, while the other basic ten-fold pattern has angles of 36°. A surprisingly floral design is given by 90° (top left), while 72° generates a pattern combining both families of ten-fold forms, an example of how successfully they can be integrated (top right). Finally, an angle of 54° produces particularly elegant petal shapes and a small central star the same shape as the first example’s central star (bottom left).

The final example opposite takes the second variation (dotted line) and replaces its rosettes with those found on the previous page, showing just how beautifully harmonious the interrelationships between patterns in these ten-fold form-languages can be.
Perfect Fourteen
number of the prophet

The patterns shown here are based on fourteen-fold rosettes, the petals of which fit within the central star in the same way that the petals on page 37 fit within their central ten-fold star. However, the proportions inherent to the heptagram and the tetradecagonal star are more complex than the unique golden section found in the pentagram and the decagonal star, and, as a result, they can easily combine to fall out of synchronization with each other. Patterns in this fourteen-fold family are thus much harder to design successfully and are consequently much rarer. Two basic patterns are shown below. The more intricate pattern shown opposite is rendered as it is found in a carpentry panel at the mausoleum of the Mamluk Sultan Qaytbay (d. 1496 C.E.) in Cairo.

In the Islamic calendar the month begins on the evening of the new crescent’s sighting, making the fourteenth of the month the night of the full moon, when the moon reflects the sun’s light most fully on the Earth (the moon can also appear full on either the thirteenth or fifteenth). Accordingly the Prophet Muhammad, held to be the mirror of Divine Light within creation, is associated with both the full moon and the number fourteen.
Singular Stellations
working with odd numbers

With a few notable exceptions, such as five and seven in ten- and fourteen-fold patterns, or multiples of three, odd numbers, particular prime numbers, are tricky to create patterns with.

A frequently used technique for making patterns with odd numbers is to set the odd-numbered motif along the edges of a square or rectangular section, half on one side and half on the other. This section can then be repeatedly reflected on all sides. A simple example of this technique is shown below; heptagonal stars forming an elegant dancing pattern.

Opposite is a more sophisticated design using nine- and eleven-pointed stars in the style of Persian cut tile work, based on a pattern devised by Jay Bonner. The subgrid for this pattern uses hendecagons and enneagons (opposite below left). It can be understood as reflecting rectangular sections (dotted center), or, alternatively, an elongated hexagonal repeat joining the centers of six hendecagons (shaded center). A similar elongated repeat hexagon can be set on the centers of six enneagons. Two ninths of a full turn, 80°, plus three elevenths of a full turn, approximately 98.2°, is very close to 180°. This allows a rhombic arrangement of two enneagons and two hendecagons (shaded right), the nine- and eleven-fold symmetries being almost imperceptibly tweaked to fit together.
The tweaking technique used on the previous page is not restricted to working with odd numbers—certain remarkable patterns aim to integrate many different numbers as accurately and beautifully as possible. Two examples of this type of pattern are shown opposite with their subgrids. A simpler combination of twelve-, eight-, and approximate five-fold geometry is shown below.

These patterns aspire to reintegrate the multiplicity of number in a harmonious unity, and the connection with harmony is more than just visual analogy. As with the previous pattern using nine- and eleven-fold stars these constructions rest on the fact that the sum of certain fractions is very close, but not equal, to other fractions. Similarly the very first challenge that the student of musical harmony faces in forming a scale is the small discrepancy between multiples and powers of the pure overtone wavelength fractions of $\frac{7}{12}$, $\frac{5}{12}$, $\frac{1}{12}$, and so on. For example, six pure whole tones, $(\frac{7}{12})^6$ (approximately 0.493), falls just short of one octave, $\frac{1}{2}$.

The kite shapes in the pattern below, bridging the space between the stars and defining small quadrilaterals where they overlap, are an example of a frequently used device that occurs in both number-combining patterns and those with one key symmetry.
Dome Geometry
the third dimension

Islamic architecture is well known for its domed structures. Many architects of these domes were content to present them undecorated, their engineering and elegant form proving sufficient for their goals. But on occasion domes were ornamented with geometric patterns. Well-known examples occur in the monuments of Mamluk Egypt and Safavid Iran – the dome illustrated opposite is from the mausoleum of Sultan Qaytbay in Cairo.

The basic method used in the geometric ornamentation of domes is to repeat sections like the segments of an orange. Stars and interconnecting pieces are placed in these segments and tweaked to fit as the width narrows towards the top (opposite top left). Many such domes resolve at the top with petal- and kite-shapes that form rosettes when viewed from above (opposite top right).

The true spherical equivalents of the regular and semiregular tilings are the divisions of a sphere that arise from the Platonic and Archimedean solids. There is no well-known evidence of artisans in the Islamic world using these uniquely spherical tilings – they seem to remain a largely unexplored possibility in Islamic design. The example below shows a spherical pattern derived from the cube and regular octahedron, based on work by Craig Kaplan.
Muqarnas

celestial cascades

Surmounting a square or rectangular structure with a dome necessitates a transitional device, and in time a distinctive solution for this, known as muqarnas, arose in Islamic architecture. Muqarnas are structured on tiered horizontal layers joined by flat and curved surfaces which articulate their descent – echoing the idea of spiritual light cascading from the Heavens to condense as crystalline matter on Earth. They are also used in niches, for example the niche, or mihrab, that marks the mosque wall facing Mecca.

Muqarnas' functions range from fulfilling structural necessity, for example transferring forces with carved stone in Egypt, Syria, or Turkey, to the purely ornamental articulation of space in the tile-clad structures assembled within the brick architecture of Iran, or the wood and plaster techniques of the Maghrib.

The design of muqarnas varies in different regions and eras. In the Maghrib a modular system based on eight-fold geometry proved perfectly at home (below). The east of the Islamic world employs muqarnas with concentric tiers around a central pole, some designs using different stars on each tier, others using stalactite forms within curved bays (opposite), and yet others emphasizing a triangular, prism-like articulation between tiers.
Traditional Islamic ornament is eminently functional – but its function is not utilitarian. It seeks to compensate for the spiritual losses of civilization by re-establishing something of the primordial beauty of virgin nature, and to transport the viewer from immersion in the mundane to serene contemplation. Islamic design can be thought of as a form of visual music; the repetition and rhythm of its motifs establish an inner sense of balance and act as a visual extension of the invocatory remembrance of the Divine.

The simplicity and apparent inevitability with which many Islamic geometric patterns unfold belie the effort involved in finding them. The anonymous artisans concerned must surely have regarded them as preexistent possibilities gifted from the Source to those who proved worthy. Not a few such craftsmen must have been well aware of the abjad equality between the words “point,” nuqta, and “geometer,” mubandis, and aspired to allow this transcendent relationship to shine through in their works.

The design opposite is based on a variation on a theme by Paul Marchant, marrying forms from the two interrelated ten-fold families. At the close of this small book it is a fitting reminder that possibilities yet remain open for exploration in the art of making Islamic geometric patterns.
The patterns below are all made of only one or two different shapes and are constructed on either a square grid or a triangular grid. They can all be drawn relatively easily on squared graph paper or isometric paper, or alternatively only require one or two stencils, making them ideal for use in the classroom. Some vertices in the square patterns lie midway between grid intersections. The two curved patterns use the compasses to trace arcs centered on, and passing through, points on their grids. The coloring schemes can vary from those shown.
An Infinite Puzzle Set

The *Einun al-Girib* is the starting point for a whole range of patterns. An excellent way to explore these patterns is to make a puzzle set of pieces. Follow the diagrams above to construct a regular decagon—a radius of about 2" is a suitable size for the first circle—and then draw each shape from this decagon, star, pentagon, "merged" double pentagon, "bottle," and "knot." Make a stencil for each shape from stiff card or thin plastic and cut out as many pieces as needed from colored card or paper to complete a puzzle set. The possibilities with such a set are limited only by the number of pieces cut, especially if one starts to explore color schemes or aperiodic and fractal repeat structures. To complete the patterns below, plus those on page 35, cut out the numbers of pieces shown. In order to leave a neat rectangular outline half and quarter shapes need to be cut as detailed. Counting, and maybe even making, the number of pieces needed for the pattern opposite, from the Purana al-Hulba Mausoleum in Agra, India, is left to the enthusiastic reader, or perhaps a classroom full of students.
Subgrids

Square Kufic

Of all the ornamental styles developed from early Kufic script, Square Kufic, set reproducibly on a square grid, is the most obviously geometric. Alphabetic writing encodes the sounds of speech and thus conveys words, sentences, new information, and, above all, meaning to the reader. Square Kufic presents a curious inversion of this function as it remains well-nigh illegible to most modern Arabic readers, its purely graphic twists, turns, simplifications, and compromises to letterform leaving them lost. It is easiest to decipher when the word or phrase written is already well known to the viewer, thus rather than conveying new information or preserving a text accurately for posterity, Square Kufic acts primarily as a talismanic invocation of sacred words and phrases already familiar. Simple words or phrases are often arranged in rotating repetitions (top row), longer passages (middle row) are arranged spiralling round from the outside, often starting at the bottom right corner (bottom row).

Gilded interlaced borders are frequently found in the Islamic arts of the book. They are used to frame central geometric panels in frontispieces, section, and chapter titles throughout a work and, on occasion, whole pages of text, particularly in illuminated Qurans.

One technique that is commonly used is to construct these braided borders on a simple grid of dots, often colored blue and red. The following selected examples are provided as an introduction and brief reference to such designs for aspiring illuminators.