Across the Islamic world, illuminating Korans from Morocco to Malaysia, and adorning mosques, mausoleums and palaces, are hidden some of the most exquisite geometrical devices ever conceived by man.

In this excellent little book, geometer Daud Sutton unravels the mystery of Islamic patterns, explaining where they come from, how to draw them, and hinting at the Divine messages they encode.
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the Infinitely Good, the All-Merciful.

This book is dedicated to Dr. Martin Lings, much loved and greatly missed.

Special thanks to Professor Keith Critchlow for opening the door and continuing to inspire, to Paul Marchant for all the teaching and ongoing support, and to Fauzid Gouverneur for years of insight and guidance.

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If you have enjoyed this book, Professor Keith Critchlow’s Islamic Patterns, published by Thames & Hudson, is highly recommended. Also recommended are Unity in Pattern by Paul Marchant, Arabic Geometrical Pattern and Design by J. Burgoin, Splendours of Qur’an Calligraphy and Illumination by Dr. Martin Lings and Arabesques: Decorative Art in Morocco by Jean-Marc Casséra.

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INTRODUCTION

The role of sacred art is to support the spiritual life of those whom it surrounds, to instill a way of perceiving the world and the subtle realities behind it. The challenge thus facing the traditional artisan is how to build with matter so as to best embody spirit. The great temples, churches and mosques of the world are the legacy of our attempts to do just this, each determined by the spiritual perspective in question.

Throughout their long history the craft traditions of the Islamic world evolved a multitude of styles applied to a great variety of media, but always with unifying factors that make them instantly recognizable. It is perhaps no surprise that an art form that seeks explicitly to explore the relationship between Unity and multiplicity should be at the same time unified yet diverse. Harmony is central.

The visual structure of Islamic design has two key aspects: calligraphy using Arabic script – one of the world’s great scribal traditions – and abstract ornamentation using a varied but remarkably integrated visual language. This art of pure ornament revolves around two poles: geometric pattern, the harmonic and symmetrical subdivision of the plane giving rise to intricately interwoven designs that speak of infinity and the omnipresent center; and idealized plant form or arabesque, spiraling tendrils, leaves, buds and flowers embodying organic life and rhythm. This book focuses on Islamic geometric patterns, exploring their structure and meaning.
Consider a point, dimensionless position in space. Extending the point defines a line (below left). Turning the span of this line about the first point traces a circle, the first and the simplest geometric plane figure and Unity's perfect symbol. Mark a second circle, centered on the circumference of the first and passing through its center. Continue by placing circles at each new intersection to fit six identical circles cycling around a central one, the ideal representation of the Quranic six days of creation. This beautifully simple construction can be continued indefinitely (opposite), defining a tessellation of regular hexagons perfectly filling the plane.

The mid-points of a regular hexagon's sides join to form a double triangle (top right opposite), known in the Islamic World as the Seal of Solomon – it is said that the ring by which he commanded the jinn bore this crest. Repeating the outline of this six-pointed star within each hexagon gives a pattern of stars and hexagons.

The final stage opposite shows the pattern as it is found in the Ibn Tulun Mosque in Cairo (879 C.E.), carved in plaster. The pattern's lines have been rendered as interlacing bands, passing over and under each other where they cross, and the remaining spaces filled with arabesque motifs.
**Sixes Extrapolated**

*some more basics*

Many different techniques of geometric construction have been used throughout the Islamic world, adding practical aids such as set squares, stencils and grids to the fundamental tools of compasses and straight edge. Most of the small selection of constructions in this book use a philosophical method relying solely on compasses and straight edge and emphasizing the geometric underpinning of the patterns as a whole.

Simple patterns lend themselves to adaptation in many ways and the constructions shown opposite develop two variations on the star-and-hexagon pattern from the previous page. The points of intersection in the patterns' paths remain fixed at the midpoints of the hexagonal subgrid's edges, while the stars expand and contract respectively. The same equilateral three-fold hexagons occur in both of these adaptations, yet the two patterns give remarkably different overall impressions.

Below is another example of how simple patterns can extrapolate into more complex ones. Starting again with the star-and-hexagon design, cut four points off some stars to form rhombs (center) and remove small hexagons to create a pattern that reads as both individual shapes and large overlapping hexagons (right).
TRANSFORMING A SUBGRID
and framing the infinite

The construction on the previous page also defines a semiregular tiling of equilateral triangles, squares, and regular hexagons (left, top row). Notice that this design, when repeated (right, top row), is itself underpinned by the regular tiling of hexagons (dotted line).

Picture the triangles in the pattern inflating as the hexagons are pinched, the squares giving exactly as much as they take. When the triangles have become the same equilateral three-fold hexagons as on the previous page, a beautiful pattern of overlapping regular dodecagons is defined (left, middle row). Continuing until the triangles form regular hexagons gives a second frequently encountered pattern (right, middle row).

Conceptually a repeating pattern can continue forever, but in practical applications Islamic patterns are generally cropped to form rectangular sections with corners in the center of key pieces, often stars (bottom row, opposite). Framing a pattern this way maintains a geometric elegance at the same time as clearly implying that it could repeat indefinitely, as it were, under its borders – the perfect visual solution to calling to mind the idea of infinity, and hence the Infinite, without any pretense of being able to truly capture such an enigmatic concept visually.

This framing also usually gives a single central piece which ensures that the total number of pieces in the rectangle is odd – a numerical quality traditionally said to invoke, and find favor with, Divine Unity.
**GIVE AND TAKE**  
*the breath of the compassionate*

Start with a circle set on a horizontal line and trace arcs centered on and passing through the intersections to define a vertical (*below left*). Repeating this on the new intersections defines diagonals on which four circles, identical to the first, can be drawn. Add four more circles to produce an array of eight around one. As with the pattern on page 3 this circular matrix can be continued indefinitely to define a tessellation, this time of squares (*opposite*).

Combining a horizontal square with a diagonal one produces an eight-pointed star (*top right opposite*). Like the double triangle this double square is known as the Seal (*khātam* in Arabic) of Solomon, for the legends vary, and is the starting point of a vast family of patterns (*see page 26*). Repeating them in each square makes the fundamental pattern of stars and crosses opposite.

This pattern can also be seen as a tiling of smaller diagonal squares, half of which expand and the other half of which contract. For this reason it has, in recent times, been referred to as *The Breath of The Compassionate*, a name referring to the teachings of the Great Master Ibn al-ʿArabi which expound the Divine Breath as the basis of creation, liberating the possibilities of the four Elements; Fire, Air, Water and Earth.
Eight-Fold Rosettes
and some construction principles

A prevalent device in Islamic geometric patterns is the distinctive geometric rosette, with its petals arranged around a central star like an archetypal crystalline flower. Rosette patterns such as these can also be seen as a network of stellar motifs, inverting perception to picture the petals as negative space. Shown here are eight-fold rosettes rendered in a style based on carpentry panels.

Two methods of construction are shown. Below is a simple one based on a square grid; here the large regular octagon is defined by diagonals and a circle, and partitioned into the geometric rosette with petals one quarter the width of the whole square repeat. Opposite is another that ensures the points of the five-pointed stars, two halves of which are set on each side of the square, all lie on the same circle. This makes the four short edges of the hexagonal petals identical in length, a geometric subtlety particularly common in carpentry applications.

The other patterns opposite show some of the ways that the shapes generated in the simple rosette can be rearranged, giving rise to new shapes in the process. Repeating sections are not restricted to squares only, but include carefully proportioned rectangles.

A variation in a rectangle with sides in the ratio 1:√2.

A large composition showing the harmonious interaction of the small octagons and the eight-fold rosettes.

The central vertical section of the pattern to the left titled as a repeat unit, two shown here.
Calligraphy
the proportioned alphabet

Quran literally means recitation, for initially the Holy Book was memorized by heart. Soon, however, it became necessary to record it in written form and the hitherto rudimentary Arabic script became the focus for generations of devoted scribes striving to develop the most suitable hands for the scripture.

The first truly Quranic script to be used (ca. 9th c. C.E.) is termed Kufic, after the town of Kufa in Iraq. Predominantly horizontal in movement, its commanding presence conveys majesty and austerity (below). Many ornamental scripts were derived from Kufic (see page 57) and remained in use long after the original hand.

The most well known styles of Arabic calligraphy today are the cursive scripts. Their refined form originated in the inspired system of proportioning developed by Ibn Muqla (d. 940 C.E.), prior to which they held a relatively low profile in relation to the majestic Kufic. Here the fundamental starting points of geometry also underpin calligraphic form – every letter is carefully proportioned in relation to the circle, its diameter, and the point, or **nuqta**, marked as a rhombic dot with the reed pen. The first and most fundamental letter is the **alif**, traced as an elegant vertical stroke within the circle. Different systems of proportioning the **alif** exist, using six, seven, or eight vertically spaced **nuqats**.
**ARABESQUE**

*the gardens of paradise*

Arabesque designs, *islāmi* in Persian, are the complement of geometric patterns. They aim not to imitate the plant kingdom naturally but to distill visually the essence of rhythm and growth it manifests, recalling the archetypal Gardens of Paradise. Varied arabesque styles (*opposite*) are one of the more obvious differences between regions and eras of Islamic design.

Spirals are primordial and universal symbols, intimately related to life and its cycles. They embody the eddying process of Creation’s expansion and contraction and find their application in Islamic design as the basis for many arabesque motifs. Designs such as the one below are often found winding behind Quranic text, in friezes and the title panels of illuminated books. When used this way the vine continues behind the letters while the leaves and flowers fill the remaining spaces.

The spiral is associated worldwide with the sun and its yearly cycle. The sun unwinds from its rebirth at the winter solstice, loops ever more widely in the sky, past the balance points of the equinox to the summer solstice, when it is sky borne for the longest period in its cycle, before winding back up to its midwinter demise.
Six of One
half a dozen of the other

Start with the basic star-and-hexagon pattern (below left) and rotate each star through one twelfth of a full turn (below center). Extend the lines of these stars' corners to form small triangles to create a basic pattern of twelve-fold stars (below right).

Illustrated opposite is the generation of a pattern based on the semiregular tiling of regular dodecagons, regular hexagons and squares. Stars are set within the subgrid's shapes, with points of sixty degrees touching the midpoints of each edge. As below, the twelve-pointed star is made from two overlapping six-fold stars.

The star points of Islamic patterns often touch to form two intersecting line segments making over-under interlacing, as used opposite, an easily applied articulation. Appropriately, geometric patterns are known in Persian as girih, literally knots, calling to mind weaving and the talismanic effect of knots and braids. A pattern with interlacing strap work no longer has mirror symmetry; reflecting it turns all the "overs" into "unders" and vice versa.

The world's spiritual traditions are in agreement that what we see of the world rests on an unseen, subtle and meaningful order. Likewise, the subgrid and implicit circles of patterns like that opposite are openly concealed in the finished design, hidden in plain view by the clothing through which they can be perceived.
THREE TIMES FOUR

and four times three

From the disciples of Christ to the months of the canonical lunar year the number twelve has many associations in Islam. Twelve is the first abundant number, a number whose factors sum to more than itself: \(1+2+3+4+6=16\). These factors also all occur in either the hexagonal or square repeat systems, making twelve-fold motifs particularly useful in pattern making.

The diagrams opposite explore the family of patterns introduced on the previous page. The basic pattern of twelve-fold stars is shown unfolding from the semiregular tiling of regular dodecagons and triangles (opposite top). Notice the way that the finished pattern can be seen as overlapping large hexagons and interwoven zigzags.

Arranging dodecagons edge to edge on a square repeat gives the second subgrid opposite. The star pattern created from this subgrid can be seen as overlapping octagons with interlaced paths.

Take the arrangement of four triangles around a square from the second subgrid and use it to space dodecagons in a triangular repeat to create the third subgrid opposite. The spaces left form the same three-fold hexagons as on page 5 (below, fourth shape).

This third subgrid can be extrapolated into a fourth, arranged in a square repeat. Adding the relevant stars creates a particularly sophisticated pattern. Note the dodecagonal paths around the twelve-pointed stars in both the third and fourth patterns.
FURTHER TWELVES

and some rosette constructions

Generating patterns by the interplay of square and hexagonal repeat structures with twelve-fold motifs is not limited to the previous examples. For example, shown opposite are a pattern from a door panel in the Christian quarter of Damascus and a variation on this by Paul Marchant. Their underlying structures are shown below: groups of three squares forming twelve-fold stars are set respectively on square (left) and triangular (right) grids – the dotted lines show the sections used in the illustrations. Dedicated geometers will note that the corners of the squares (marked opposite as black dots) define lines that give the rest of the points needed to develop the full pattern (white dots). These lines also extend to give the proportion of the central stars within the rosettes.

The same rosette proportions can be established independently of the pattern, as depicted opposite with two more rosette constructions. All three constructions start from a double hexagon set within a regular dodecagon with radial lines added. Black dots mark key points that are found in the initial structure, gray dots mark any intermediary points needed, and white dots mark the points that give the final proportions of the star.

Source pattern from Damascus with square repeat above. Triangular repeat variation opposite.

The rosette used above, proportioned from its outer structure.
A rosette constructed to make the petals' four short edges equal.
Narrow petals with an additional harmonious star added.

The rosettes above, plus a fourth variation, placed within a square repeat framework from Alhambra.
THREE-FOLD PERMUTATIONS
multiples from the matrix

Most of the patterns covered so far repeat on either a regular hexagonal or square grid, and a more systematic look at the hexagonal grid is shown below. Joining the centers of the hexagons defines a regular tiling of equilateral triangles — these two grids are each other’s duals. Any pattern that can be repeated with regular hexagons can also be repeated with equilateral triangles.

The smallest section needed to define an entire hexagonal repeat pattern is one of the light gray or white triangles below. These triangles have sides in the ratio $1:\sqrt{3}:2$ ($\sqrt{3}$ is approximately 1.732) and this structure is sometimes called the $\sqrt{3}$ system. By rotating, reflecting, and translating (sliding) one of these triangles it is possible to generate the entire pattern. Some traditional methods construct a stencil of such a triangle and apply these three symmetry movements to complete the pattern.

The points where three hexagonal repeats meet have shapes with rotational symmetry in multiples of three, and the points where six triangular repeats meet have shapes with rotational symmetry in multiples of six (opposite top left). These multiples permute to give different numbers at these key points. Each illustration opposite shows the same portion of a pattern in relation to the subgrid.

A 1:3:2 rectangular section made of 1 hexagon and 4 quarter hexagons, or 2 triangles and 4 half triangles.

A Seljuk design with wide scoop work. Threes and sixes, in this case triangles and six-pointed stars, lie on the key points.

A Seeing with wide scoop work. Threes and sixes, in this case triangles and six-pointed stars, lie on the key points.

A pattern combining fifteen, five times three, with twelve, two times six. For nine and twelve see the cover of this book.
FOUR-FOLD PERMUTATIONS
quadruples in quadrangles

Joining the centers of squares set in a regular tiling defines another square tiling – the square grid is its own dual. The smallest section needed to define a whole square repeat pattern is one of the light gray or white triangles below. These triangles have sides in the ratio 1:1:√2 (√2 is approximately equal to 1.414) and this structure is sometimes known as the √2 system. As with the hexagonal system, rotating, reflecting, and translating this minimum section can generate an entire pattern.

Two minimum triangles, long edge to long edge, form a square and with so many squares to choose from it can sometimes be tricky to distinguish the two dual grids and the minimum section triangles in a square repeat pattern. In addition, the size of a pattern’s pieces relative to the repeat can vary considerably. However, with a little practice these structures can be discerned quite easily.

In square repeat patterns the points where four squares meet have shapes with rotational symmetry in multiples of four, but, as there are two dual square grids to consider, different multiples can be combined (opposite top left). Opposite are some of the ways these multiples permute, each illustration showing the same portion of a pattern in relation to the subgrid.
PIECES OF EIGHT

barbary brilliance

Over the centuries artisans in the Maghrib, the West of the Islamic world, explored a remarkable form-language based on the square repeat and, in particular, the possibilities inherent in the eight-fold khāram. The Breath of the Compassionate is shown below unfolding from the semiregular tiling of octagons and squares. Tracing octagram stars by joining every third corner within each khāram creates a simple pattern using this set of shapes.

Opposite is an example of the way simple geometric relationships build upon each other from an initial square grid to generate a whole series of different square repeat patterns. The shapes that arise from generating these patterns are collected in the central panel. Together with many other related shapes they are cut to this day in Morocco in brightly colored friable tiles to create a vast puzzle set with countless solutions.

The two-fold hexagon found third down in the left hand column of the central panel opposite, known as a safi, is a particularly important shape in this system. The safi plays a fundamental role in the generation of more complex zillij patterns such as the one covered on the following page.
**Zillij Design**

*eight-fold extravaganzas*

The cut tile work of the Maghrib is known as *zillij*. The medieval glaze palette for this work was limited to only a few colors; black, white, dark green, turquoise, blue, and a warm yellow ochre. Nowadays many other colors are also used.

Vast compositions can be made with *zillij*, compiled in a modular technique that alternates *khattam* with *ṣafī* to make a framework of sections to fill (*black pieces opposite*). Rings of color are arranged within these sections to create designs that read well to the eye from far away, when geometric detail can no longer be seen, and close up, where individual shapes are clear. Note the way that the same shape occurs in different tones in the design.

The rosettes used opposite have twenty-four and sixteen petals, requiring the construction of pieces that interface with the eight-fold geometry. Although asymmetric these pieces read comfortably in context, perhaps because of their geometric necessity.

*Zillij* can be composed on paper using a simple square grid (*below*), sketching approximations of the forms before assembling the final work with correctly cut tile pieces. This method works by replacing the $\sqrt{2}$ ratio of a square’s diagonal to its side, found in the correct shapes, with the fractions $\frac{2}{5}$ (1.5) and $\frac{3}{2}$ (1.4).
Self-Similarity

the same at different scales

Mathematicians use the term *fractal* to refer to mathematical objects that exhibit self-similarity, with the same forms and patterns occurring at many different scales. Fractals embody infinity through the endless recurrence of similar structures, rather than the unbounded continuation of repeating pattern. This concept has long been used in some aspects of Islamic design.

The panel opposite is based on *zillij* found in the Alcázar in Seville. A complex web of interlaced white strap work contains the familiar *zillij* shapes in blue, green, ochre and black. Remarkably, these shapes then form large versions of themselves, outlined in black. This design also contains a third level of implicit self-similarity – the interlacing strap work is proportioned exactly as if it too rested on even smaller *zillij* pieces (opposite below). It seems the designers of this pattern were well aware of the possibility of this subdivision continuing indefinitely.

Self-similar designs such as this are not limited to eight-fold *zillij*; the families of forms derived from ten-fold geometry (pages 34 to 37) are also eminently suited to this type of composition. Self-similarity also occurs in arabesque designs, with leaf forms composed of interconnected smaller leaves and vines (below).
Arcing Patterns
the balance of line and curve

Not all Islamic patterns leave circles hidden within their implicit construction. Geometric designs that combine arcs and straight lines in their final forms have been a feature of the art form since its beginning. They are usually found rendered in materials that are relatively easy to form into curved shapes, such as the painted arts of the book, metalwork, and carved stone. Patterns using arcs have a distinctly softer appeal, on occasion giving the impression of merging with the arabesque designs their pieces may contain.

The pattern below is from a carved stone window grille in the great Umayyad mosque of Damascus (715 C.E.). Straight bands form the semiregular tiling of regular hexagons and equilateral triangles. Interlaced with these are sections of circles centered on the triangles' vertices and passing two thirds of the way along their sides.

The design opposite is based on a pattern given to Professor Keith Critchlow. The arabesque motifs filling the spaces are in the style of Mamluk Quran illumination. These two patterns are an excellent example of the way that the subgrids used, often quite obvious in early patterns, are more fully concealed in later work.