theory which he has already put forward in his Optics to refute - in the lost part of On the Light of the Moon, where he considers different shapes of the moon - a possible objection to the thesis of the sphericity of the moon. But given the fact that both Ibn al-Haytham’s doctrine concerning the emission of the light of the moon and his theory of perception are original and have no precedents in the history of thought in Islam, it is extremely unlikely that the author of the common source of al-Birjandi and al-‘Ubaydi (Fakhr al-Din al-Rāzī or anyone else) should not only arrive at the same theory concerning the emission of the light of the moon, but also reproduce part of his theory of perception. Thus, we are led almost naturally to the conclusion that both al-Birjandi and al-‘Ubaydi’s discussions about the configurations which would have produced the phases of the moon, as well as the latter’s discussion of the possible shapes of the moon and his physical explanation of the apparently flat shape of the moon, all go back, directly or indirectly, to Ibn al-Haytham and his treatise On the Light of the Moon.

A final stylistic remark. Ibn al-Haytham was very fond of the word المستفاغات in the sense of ‘too far away’. But to al-‘Ubaydi, writing about three centuries after him and maybe from a different linguistic standpoint, this usage of the word المستفاغات looked somewhat strange. That is why he took it to be a technical term and tried to make it more understandable by using the phrase “البعد الكبير المسمى مستفاغاتا”.

In this paper I have tried to show that when dealing with scientific texts it is sometimes necessary to consult not only the different manuscripts of the text in question but also other manuscripts somehow related to it. It is this cluster of manuscripts which helps us to locate and reconstruct the missing part of the text. But to engage in such research the basic problem of the text and its scientific content have to be isolated. This is an effort which goes far beyond merely textual considerations.

THE SPECIFIC CASE OF GEOMETRICAL MANUSCRIPTS USING THE EXAMPLE OF MANUSCRIPT B.N. 2457 (PARIS)

Hélène Bellosta

This paper aims to study some specifics of geometrical manuscripts through the study of the very famous manuscript No. 2457 from the Bibliothèque Nationale in Paris. If we put aside the intrinsic difficulties of a mathematical text, which pose problems for a reader who is not a mathematician and even sometimes for a mathematician, geometrical manuscripts offer two advantages over other kinds of manuscripts as regards the edition of the text. First, on account of their inner logic they offer the reader who is a mathematician the possibility of checking, or even restoring, the inner consistency of the text, and of detecting its lacunas. Secondly, the presence of geometrical figures illustrating and supporting the demonstration enables the reader to understand it better (consult Descartes for whom the figure is here “pour fixer l’imagination”); these figures can also help the editor of a geometrical text to establish or confirm a stemma. Many elements have then to be considered concerning these figures, their place in the text, and their nature: were they done at the same time as the copy or later, with the same ink or with a different one, or, what is more difficult to assess, are they by the same hand or a different one?

As is very well known, the most important part of manuscript No. 2457 has been copied by the tenth-century mathematician al-Siǧzi. This raises some other questions: did the copyist’s intellectual background influence the quality of the copy. More precisely, does a mathematical text copied by a mathematician have particular characteristics, and if it does, what
are they? And last but not least, what kind of reasons drive a mathematician to copy a text? There may of course be financial need: al-Qifṭī tells us, for example, that Ibn al-Haytham, at the end of his life, would copy scientific texts (Euclid, Ptolemy’s Almagest, The little astronomies) for a wage of 150 Egyptian dinars. If the motives of the mathematician as a copyist are due to financial need, we can expect him to behave like a real copyist, being more or less meticulous, according to his mood. But if his motives are intellectual, he may be tempted to keep the substantifique moelee of the text, giving us therefore not a full copy, but an edition of it (تحرير تفسير).

This is the case with the thirteenth-century mathematician and astronomer Naṣīr al-Dīn al-Ṭūsī3 who gave us a great number of editions of mathematical texts. The mathematician as a copyist may also be tempted to comment on the text he is copying, giving us not a copy but a commentary of it (تفسير تقسيم). He could also be tempted to correct what he supposes to be errors in the proofs, either with appropriate (hence undetectable) corrections, or inappropriate ones if he misunderstood the demonstration. Both of these corrections complicate the task of establishing a stemma. The three stages of a text we have just mentioned - copy (نسخة), edition (تحرير), and commentary (تفسير) - are all interesting for a science historian, since they allow him to appreciate the role of a copy and to distinguish between a dead text (which has become an object in a museum of the history of science) and a text which is still alive and used as working material by mathematicians; nevertheless, he must be careful not to confuse them.

Now let’s get to the point of manuscript No. 2457 from the Bibliothèque Nationale in Paris. This manuscript was brought back from Cairo at the beginning of the XIXth century, by a certain Reiche, a student of Caussin de Perceval. It is a collection of about 50 treatises or opuscules, most of them dealing with mathematics. The importance of this compendium was not lost on historians of mathematics: the list of its treatises was published for the first time by Woepcke4, and then again by de Slane; it was described by G. Vajda, and was most recently the subject of a paper by P. Kunitzsch and R. Lorch. Besides some autograph texts of al-Sijzi, this compendium contains, among others, many texts of Thābit Ibn Qurra, and two texts of his grandson Ibrahīm Ibn Sinān. Almost all the writings of these two mathematicians concerning ‘infinitesimal mathematics’ are to be found in this compendium, as well as a very important treatise of Ibn Sinān on Analysis and synthesis5, and two geometrical opuscules of Abū Sahl al-Qūhī6. It also contains a treatise of Ibn Sahl dealing with the ‘three conical sections’7.


8 P. Crozet is editing al-Sijzi’s works.

9 I.e.: The measurement of paraboloïds (ff 95v-121r), and The measurement of the paraboloid (ff. 122v-134v line 13) by Thābit Ibn Qurra, and The measurement of the parabola (ff. 122v line 14-134v line 4) by Ibn Sinān. R. Rashed, Les mathématiques infinitésimales, vol. 1.


Moreover, this compendium gives the only version known today of some of these treatises, and they are important ones. This confirms, if necessary, the importance of this collection for the historian of mathematics.

Manuscript No. 2457 is a small, leather-bound volume of 220 folios, written on dark brown paper, with blackish brown ink. Some pages are badly stained, and the first folios are in a very poor state. The lower corner on the binding side of the first thirteen folios has been very carefully repaired with a slightly lighter paper, the damaged parts being copied again, where possible, in a different hand. However, the restorer left a blank for six passages that he probably could not read and for some figures he could not complete (figures 1, 4, 6, and 9 of Ibn Sinā’s treatise on *Analysis and synthesis*). This restoration also explains the origin of some errors (8 words). The pagination of the manuscript is in western numerals, the first folio (A) bearing no number; there is also a far less legible and older pagination in *abjad* numerals, different from the previous one. Folios 16 and 25 have been inverted, and some folios have been bound upside down, probably when the manuscript was bound for the last time. The last folios are written on a different, stiffer paper. The first 192 folios, that is, texts number 1 to 44 (with Woepcke’s numbering), are by the same hand (al-Sīğzī’s). The other folios are by different hands, some of them very similar to al-Sīğzī’s.

As regards the colophones, these texts were for the most part copied by the tenth-century mathematician al-Sīğzī in Shirāz between 358-361 AH (969-972 AD). It is worth noting here a peculiar aspect of this dating: while seven of these texts are dated according to the Hijra, four of them are dated according to the Yazdegerd era. In Persia, a dating according to two Yazdegerd eras can be found, mostly used by the Zoroastrians and also by astronomers, one of them dating from Yazdegerd III’s accession to the throne (632 AD), the other one from his death (651 AD). The coherence of dating leads us here to use the first Yazdegerd era, the one dating from 632: the years are of course solar ones, and the months are the months of the present Persian calendar. Let us consider the three texts dealing with infinitesimal mathematics which were copied sequentially in Shirāz, that is, texts 24, 25, and 26 (according to Woepcke’s numbering) or 1, 2, and 3 (according to Kunitzsch and Lorch):

*The measurement of paraboloids* by Thābit Ibn Qurra, ff. 95v-122r, dated Rabī’ al-akhir 358 AH, and more precisely ten days before the end of this month, i.e. 13.2.969;

*The measurement of the parabola* by Thābit Ibn Qurra, ff. 122v-134v line 13, undated;

*The measurement of the parabola* by Ibn Sinā, ff. 122v line 14-134v line 4, dated Ordibehest 338Y (April-May 969 AD).

Why did al-Sīğzī, who copied these three texts one after the other within a period of four months, change the dating between the first and the third texts? It was recently asserted that this manuscript was a copy made in the thirteenth century of a manuscript of al-Sīğzī. If this hypothesis were true, it would

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13 This is the case, for example, with Thābit Ibn Qurra’s treatise on *The measurement of paraboloids*.

14 See Kunitzsch and Lorch’s paper which studies this pagination in detail.


15 Texts 1, 6, 14, 15, 24, 32 and 41, with Woepcke’s numbering.

16 Texts 26, 27, 28 and 38, with Woepcke’s numbering.
mean that the thirteenth-century copyist would have copied al-Sīgzi’s compendium as it was, including the colophons, which was not the way of copyists. As long as no stronger argument supports this thesis, we shall stick to the conclusions of scholars (especially Vajda who studied this manuscript very carefully - its paper, ink, and handwriting), and we shall adopt the conclusion of Kunitzsch and Lorch’s paper: “we conclude ... that there is no reason to doubt the authenticity of the colophons or of Sīgzi’s writing of the copy.” The fact that the most important part of this manuscript was copied by al-Sīgzi leads us then to another point: for whom and for what reasons did al-Sīgzi copy these texts? My own feeling, based on the lack of aesthetic concern in the titles and the handwriting, and on the fact that some texts are copied one after the other - the beginning of one being on the same page, or even the same line, as the end of the other (remember that paper was a rare and expensive material) - is that he copied these texts for himself; this would also explain some peculiarities of the figures. This answer to the first question would help us to answer the second one - why did he copy them? We can imagine that, from the texts that he could get, he copied those that interested him. Indeed, the content of this compendium reveals a certain homogeneity: the fact, for example, that the three treatises dealing with infinitesimal mathematics were copied one after the other is not, in my opinion, pure chance at all.

to be attributed to a mathematician of the level of al-Sīgzi (he does not say to what text he alludes). We shall come back to this argument in the second part of this paper.

His last argument is that al-Sīgzi’s treatise On the measurement of spheres by spheres (مساحة الأكتر بالأكتر) ends by “this is the end of what he wrote on this book ” (آخر ما كتب من هذا الكتاب). That authors or copyists, not in the middle of the text but at the end of it, speak of themselves in the third person, seems to be a rather common practice (and a mark of modesty, as R. Rashed noted).


There is another fact which is not systematic or seen frequently among the copyists: al-Sīgzi quotes his sources when possible. We know from the colophons that some of these texts were copied on manuscripts belonging to Nazif Ibn Yomn, a Christian physician with whom he kept up a correspondence (من نسخة نظيفة بن يمن النحى). Another one was amended with the help of a manuscript belonging to a Soleiman Ibn’Asma (عمران نسخة كانت بخط سليمان بن عصمة). Ibn Sinān’s treatise On the measurement of the parabola was also amended with the help of another manuscript (عمران نسخة أخرى غريبة بنفسى هذه المقالة); al-Sīgzi mentions too that other texts were collated with the autograph manuscript (عمران بالأصل).

Let us now study some specific characteristics of two treatises of this compendium more precisely - Thābit Ibn Qurra’s treatise On the measurement of paraboloids, ff. 95v-122r20, and Ibrāhīm Ibn Sinān’s treatise On analysis and synthesis, ff. Av-18v21.

Thābit Ibn Qurra’s treatise On the measurement of paraboloids shows a very noteworthy peculiarity that R. Rashed did not fail to notice22. Three folios (110v-113r) have been copied twice. This accident could at first sight cast doubt on the meticulousness, of al-Sīgzi as a copyist, and could lead us to wonder if he understood the demonstrations he was copying. The repetition is, however, found in a proof which is one of the longest and most difficult of those inherited from Arabic

20 It is noteworthy that الأصل does not always refer to the autograph manuscript and may simply refer to the source, whatever it is, from which the manuscript was copied. For example, this is the case with manuscript Damas Zahrāyi 5648, which contains Ibn Sinān’s treatise On analysis and synthesis, copied on manuscript al-Qāhira, Dar al-Kutub, Riyāda No. 40, in which (folio 119v) the copyist writes بالعربية بالاصلي where the copyist of the al-Qāhira manuscript left a blank, probably because he could not read some words. Yet, in manuscript B.N. 2457, regarding the way the other colophons are written, الأصل always refers to the autograph manuscript.


22 Ibrāhīm Ibn Sinān, Mathématiques, astronomie et logique.

mathematicians (no less than 35 lemmas). To quote what the tenth-century geometer al-Qhāfi says of it in the preface to his own proof of the paraboloid volume:

وَلَا نَظَرَنا فِي هَذَا أَسْهِلْ عَلَيْنا جَدًّا، وَكَانَ كَتَابُ أَرْشَمِيسِ فِي الْقَرْءِ وَالْأَسْلَمَانَ مَعَ صُمْوُبِهِ وَكَثُرةِ أَغْرَاضِهَا أَسْهِلْ عَلَيْنا مِنْ هَذَا الْفَرْغُ الْوَعِيدِ، وَوَضْنَا أَنَّ حَالًا فُلُوْنِ في هَذَا الْكَتَابِ مِنْذُ الْمَوْتِ الَّذِي أَفْقَهَ ثَابَتُ بِقَرْءٍ في وَهَلَّ الْكَثِّرَةِ فِي هَلْقَةٍ عَلَيْنَا، فَأَقْتَضَامَا ذَلِكَ أَنْ جَدُّهُمَا "الْفَنَّرَفِ" إِسْتَخْرَاجُ مُسَاحَةُ هَذَا الْجُسْمٍ أَعَنَى المَكَائِنَ، ابْدَأَ ابْدَأً.

I suggest that this kind of mistake is not that of a professional copyist. In any case, the repetition gives us a second copy of this passage, which is important because manuscript No. 2457 gives us the only version of the text known today, and allows us to appraise the meticulousness of al-Ṣiğ̣ẓi as a copyist. In the first copy, when compared to the second one, we find the omission of one sentence, the repetition of one sentence and one word, and five errors. When the second copy is compared to the first, we find one amendment made in the first copy and not in the second, the omission of two sentences and one word, and four repeated sentences.

Ibrāhīm Ibn Sinān’s treatise On analysis and synthesis was copied by al-Ṣiğ̣ẓi in Shirāz during the month of Rabi‘ al-akhir in the year 358 AH (969 AD). Contrary to his habit in other texts he does not mention it, so we may assume that this text was not collated with the autograph manuscript. The handwriting is of a naskhī style, very often without any diacritical dots, especially on the initial letters of verbs; three points separate the paragraphs. Very few deletions are found in the manuscript (thirteen words or expressions are crossed out). It was read again and amended (33 additions or amendments are found in the margins of the text). The usual symbol marking the place of these amendments is found in the text. However, ten of these symbols appear in the margins and in the text without any amendment, which shows that the second part of the text was read less carefully than the first part.

The figures inserted in the text are drawn in red ink, the letters on the figures being of the same ink as the text. If we compare the three manuscripts used to establish this text (Paris B.N. 2457, ff. Av-18v, denoted P; Bankipore 2468, ff. 21r-39v, 631 AH, denoted B, and Cairo Dar al-Kutub, Riyāḍa No. 40, ff. 130v-153v, 1159 AH, denoted Q, all of which belong to largely independent families, that is, they have not been copied from the same source) we notice that the figures in manuscript No. 2457 are less carefully drawn but more accurate. The ones with more faults are those of manuscript B, which confirms the impression we get from the study of the text itself. One aspect of manuscript No. 2457 is noteworthy: it contains one figure more than the others. Rather than lacunae in the other two manuscripts (which belong to different families) this figure could be an addition made by al-Ṣiğ̣ẓi as a mathematician. The additional figure is to be found in the proof of problem 21, with which Ibn Sinān exemplifies the three parts of a proof - analysis, synthesis, and the search for the number of solutions. These three parts of a proof are illustrated in manuscript No. 2457 by three (two in the other manuscripts) very similar and very simple figures, not strictly necessary to the understanding of the proof. The notable errors concerning the figures drawn by al-Ṣiğ̣ẓi are that besides a very rough error bearing on parallelism (unworthy of a mathematician) he usually shows some disregard for equalities or inequalities of length implied by the text. It may be paradoxical, but we see here the hand of a mathematician, aware that proofs are not as dependent on the different configurations as they might seem to be at first sight.

As for the placing of the figures, each copyist has his own style but he usually puts them at the end of the proof they refer to. This is the case here, the only exception being figure 3 which is found at the beginning of the proof and which actually does not matter. Surprisingly enough, however, three figures (2, 5 and 6) are further away in the text, long after the end of the proof they refer to; in fact they are in the middle or even at the end of the proof of the following problem. If figure 2 is drawn on the same page as the end of the proof it refers to, figure 5 is located at the recto of this proof, in the very place where figure 6 should be, which forced the copyist to draw figure 6 beyond the end of the proof, at the foot of the same page (this discrepancy is not found in any other extant copy of this text). Perhaps here again, we are looking at the work of a mathematician who, seized by the inner logic of the proof, forgets to copy the figure and fills this gap when he becomes aware of it (especially if he copies the text for himself), rather than the hand of a servile copyist who scrupulously copies the text and figures as they are. (For an example of this kind of servile copy, look at Sharaf al-Din al-Tusi’s treatise *On equations*, edited by R. Rashed: one of the copyists, unaware of the absurdity of his work, inserted each line of a table of numbers into the text in exactly the same place it occupied in the model he was copying.)

Let us now examine the errors of copying which bear on the mathematical reasoning. My example will be problem 20, which plays a central role in Ibn Sina’s treatise *On analysis and synthesis* since it allows him to support his theory of analysis (Ibn Sina gives us no fewer than three proofs of this problem). A systematical comparison of the three manuscripts used to establish the text (Paris, denoted P; Bankipore, denoted B; Cairo, denoted Q) gives us the following results:

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>B</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Omissions &gt; 3 words</td>
<td>2</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Omissions &lt; 2 words</td>
<td>0</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>False reasoning</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Lettering errors</td>
<td>4</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Confusions</td>
<td>8</td>
<td>5</td>
<td>12</td>
</tr>
</tbody>
</table>

Three errors are to be found in all three manuscripts and may be traced back to the original.

Even if we take into account the fact that al-Sijzi was dependent on his source and on the errors found in it (and the same applies to the two other copyists) this table leads us to a rather disappointing conclusion. We have here a rather good copy, but without any distinctive characteristics; not so bad that it would be impossible to ascribe it to a mathematician, nor distinctly better than the other two which were copied by scribes who were not mathematicians. Maybe it was foreseeable: it does not seem possible, even for a mathematician, at least in the case of non-elementary proofs, to copy the text and at the same time understand the demonstrations.

What distinguishes al-Sijzi from other copyists is the off-hand way he draws figures, and the fact that he quotes his sources whenever possible. This concern of a mathematician conscious of the importance of the quality of the manuscript reminds us of the methodology of the traditional narrators of *Hadith* on the basis of which a *Hadith* was authenticated.

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