CHAPTER 4
The scribes in the administration refrain, however, from using Indian numerals because they require the use of paper; they think that the system of finger-reckoning, which calls for no materials and which a man can use without any instrument apart from one of his limbs, is more appropriate in ensuring secrecy and more in keeping with their dignity. —Al-Suli, Adab al-kutub (Handbook for Secretaries)

The spread of paper from Baghdad, the Abbasid capital, in the late eighth and early ninth centuries coincided not only with expanded literary production on many subjects and an absolute increase in the number of books but also with the development of several systems of notation. The attempt to create graphic analogues of human activities was in itself nothing new in human history, but the introduction of paper to the Islamic lands offered new possibilities for expanding the application of older systems of notation and developing new ones. Some of the systems, particularly mathematical, commercial, and geographical notation, came to be widely adopted, whereas others, such as culinary and musical notation, remain historical curiosities. Scholars have long recognized most of them individually as major achievements of intellectual life under the Abbasids, but these achievements were not accidental. Rather, they were tied to the introduction of paper: they were a product of both increased intellectual curiosity—itself fostered by the growth of learning made possible by the explosion of books—and attempts to exploit the potential applications of paper.

Why develop systems of notation? There are several motivations. Notation aids memory. It expands communication with others, allowing someone to compose a message and send it elsewhere in time or space. It allows access to a far greater repertory of ideas than a person might otherwise be able to retain in memory alone. It provides a framework for improvisation and allows the relations between parts to be visualized in new ways. Basically, it enables conceptualization at a level of sophistication that is impossible in a tradition based on memory alone.

MATHEMATICS

We instinctively reach for pencil and paper whenever we are faced with a mathematical problem, yet during the early years of Islam—as in antiquity—all people did their arithmetic mentally. When the sums became too large or the problems too complex to keep in memory, reckoners could hold intermediary results by various means. They could represent the numbers by using counters, by positioning the fingers, or by employing an abacus or dustboard. Whatever
the means of calculation, once writing was invented the results could be written
either in words ("five") or, somewhat later, in numerals (V or 5).

The use of counters—whether notches on a stick, pebbles in a pile, or sim-
ilar tokens—appears to be as old as counting itself. Archaeologists have dis-
covered bones and sticks, some more than twenty thousand years old, cut with
notches that they regard as the earliest known representations of numerals.
They have also found small clay objects bearing stylus marks and seals at many
Middle Eastern sites. Dated as early as 8000 B.C.E., these objects may have
been the simple counters used by early farmers and herders to count their har-
vests and flocks. Pebbles have also been used for numerical recordkeeping for
millennia; the English word calculate derives from the Latin calculus, referring to
a small stone used for reckoning. The pre-Islamic Arabs also counted by
means of pebbles, which they called hasa. From hasa derives the Arabic verb ihisa,
"to count or enumerate."

The ancient Mesopotamians used several different systems of counting
during the protoliterate period but eventually came to use their cuneiform
writing system to express a sexagesimal (base 60) number system, which per-
sists to this day in our division of a circle into 360 degrees. The Egyptians, in
contrast, used a decimal (base 10) system, which they expressed in hieroglyphics.
The ancient Greeks also used a decimal system, which they initially represented
by an acrophonic system of numerals; that is, they used the first letter of the
words pente (π), deka (δ), hekaton (η), khilioi (χ), and myrioi (μ) to represent five,
ten, one hundred, one thousand, and ten thousand, respectively. Then they
developed an easier alphabetic system, which gave a numeric value to each of
the letters of the alphabet, corresponding to the three series of one to nine,
ten to ninety, and one hundred to nine hundred (α = 1, β = 2, γ = 3, etc.; ι =
10, κ = 20, λ = 30, etc.; π = 100, σ = 200, τ = 300, etc.). Following the Greek
example, those who used other alphabets, eventually including writers of Hebrew,
Gothic, Georgian, Syriac, and Arabic, also began to use letters as numerals.

The equivalent system of employing the letters of the Arabic alphabet as
numerals is known as abjad, after the series of mnemonic nonsense words—
abjad hawwas, hutiq, kalamin, and so on—that reminds the reckoner that the letter
alif stands for 1, ba for 2, jim for 3, dal for 4. This system, however, created many
problems because several Arabic letters share the same letter shape and are
differentiated only by diacritical points, which are often omitted in writing.
Thus, the letters jim, ha, and kha (which in abjad represent 3, 8, and 700) all
share the same shape, although the first and last letters are distinguished from
the middle letter, which has no points, by points respectively above and below
the letter. Eventually abandoned in favor of other systems for general use, the
abjad system remained popular for specific purposes: marking scientific
instruments, such as astrolabes, numbering verses in manuscripts of the Koran, carrying out divinatory practices and composing talismans, and making versified chronograms, in which the sum of the values of the letters in a word or phrase gives the date of some event. Even in modern times abjad numbers are sometimes used for the pagination of the front matter in books, where American and European typographers would use Roman numerals.

Perhaps the most familiar ancient system of representing numbers is that of the Romans, which combines features of several systems. The numerals I, V, and X (representing 1, 5, and 10) are not letters but symbols derived from counting with the fingers and hands. Higher values are made by adding the numerals like counters (III = 3; XX = 20) or even subtracting them (IV = 4; IX = 9). In contrast, the larger numerals (C for 100; M for 1000) appear at first to be acrophonic (centum, mille), as in the early Greek system. Neither L (50) nor D (500) are acrophonic, however, and because the larger numerals were at first represented in other ways, the system must have had other—as yet undetermined—origins.

It is commonly said that Roman numerals are relatively easy to add and subtract and impossible to multiply and divide. For big tasks, virtually all ancient peoples turned to some form of the abacus, which was originally a flat surface ruled or slotted with columns in which to place calculi, identical counters used to represent numbers. (The familiar Oriental type of abacus with beads strung on rods or wires is a relatively late invention.) The abacus made it possible to do multiplication and division using Roman numerals by translating them into a decimal place value array. The advantages of the abacus meant that it was used throughout ancient and medieval Eurasia. Although it is now commonly identified with Japan, the abacus was introduced there only in the sixteenth century, and it remained popular and competitive despite the invention of the electronic calculator.

Dactylyonomy, or expressing numbers by the position of the fingers, was also known in classical antiquity, although its origins are obscure. The Greco-Roman author Plutarch, in his Lives, mentions the practice as being used in Persia in the first centuries of the Common Era, so the origins of the system may lie in Iran. The Arabs called it both the "reckoning of the Greeks and the Arabs" (hisab al-rum wa'l-arab) and the "arithmetic of the knots [finger joints]."

Several verses of the Koran seem to refer to the practice of counting on the fingers, suggesting that it was already used in Arabia before the coming of Islam. Early Muslims described or interpreted certain gestures made by the prophet Muhammad as indicating numbers, although the traditional accounts do not agree with later practice. Muhammad is said, for example, to have extended his right forefinger while pronouncing his profession of faith—"I
bear witness that there is no god but God and that Muhammad is his prophet”—because the extended forefinger means "one," signifying God’s unity.

Finger reckoning was widely used in the medieval Islamic lands. The polymath al-Jahiz advised schoolmasters to teach the "arithmetic of the knots," which he placed among the five methods of human expression as the one needing "neither spoken word nor writing." Similarly, al-Suli, in his Handbook for Secretaries, wrote that scribes working in the administration preferred dactylyonomy to any other system because it required neither materials nor an instrument, apart from a limb. Furthermore, it ensured secrecy and was more in keeping with the dignity of the scribe’s profession.

Books dealing with dactylyonomy, such as a treatise by the mathematician Abu’l-Wafa al-Buzajani, gave rules for performing even complex arithmetical operations using the fingers, such as the approximate determination of square roots. The mathematical operations were performed mentally and the fingers held in certain positions to retain partial results obtained in the process of reaching the final solution of a problem. Because of these features this arithmetic came to be known as hand or mental ("air") reckoning.

Dactylyonomy was also used in early medieval Europe, for the system described in various Muslim treatises is curiously reminiscent of that expounded in the seventh century by the Venerable Bede, the Anglo-Saxon theologian, in the first chapter of his De temporum ratione, entitled "De computa vel loquela digitorum." Using Bede’s technique a person could express and calculate with numbers between 1 and 9,999, although it was seldom used for values with more than two digits. Bede’s system, however, seems to have fallen into disuse in the West after the early Middle Ages, although a similar system continued to be used in the Islamic lands until modern times. Arab or Persian poets of the classical period could allude to someone’s lack of generosity by saying that the person’s hand made ninety-three (a closed fist, the sign of avarice). The system was still current in Ilkhanid Iran, for the fourteenth-century historian and finance minister Hamdullah Mustawfi credits the great Ibn Sina (Avicenna) with having invented calculation by dactylyonomy in 1029. According to Mustawfi, accountants were thereby freed from the bother of using cumbersome counters.

A form of dactylyonomy was still practiced until recent times in Algeria, along the Red Sea coast, and on the island of Bahrain. There the system was modified for commercial transactions involving pearls or other rare and costly merchandise. Because the buyer and the seller did not wish to reveal to bystanders the terms of the transaction, the two negotiators, sitting face to face, hid their right hands under a cloth and touched each other’s fingers according to a precise code. Although the system did not distinguish between
ones, tens, hundreds, and thousands, the parties involved in the transaction knew which sum was meant. A kind of finger reckoning even persists to this day in the United States. In the second grade my daughter learned how to use her fingers to multiply by nines. To multiply seven by nine, for example, she held up her two hands and counted off seven fingers from left to right, bending the seventh finger down: the six fingers to the left of the bent finger represented the tens, and the three fingers on the right represented the units, giving the result of sixty-three.

Although fingers are always available, calculation with an abacus or written numerals offers the great advantage that the calculator can stop work in midstream and resume later or check the work so far. The Latin word abacus actually derives from the Greek word abakos, which itself comes from the Hebrew word abaq (dust). In postbiblical usage, abaq meant "sand used as a writing surface," suggesting that the original form of the abacus was neither the familiar beads on wires nor the grooved or ruled board used with counters in medieval times, but rather a dustboard, that is, a board or slab spread with a fine layer of sand or dust in which designs, letters, or numerals might be traced and then quickly erased with a swipe of the hand or a rag. This type of temporary writing surface was known to the Babylonians and was commonly used by astrologers even in Islamic times to cast horoscopes. Apart from the simplest operations—such as adding tick marks—calculation on a dustboard, however, makes sense only when the calculator uses numerals, and the introduction of the Indian system of numerals to the Arab world made it even more advantageous.

The earliest reference in the Mediterranean world to the Indian system of numeration dates from the mid-seventh century, just after the rise of Islam. In a fragment, dated 662, of a work by Severus Sebokht, the learned bishop of the monastery of Quriasrin (located on the Euphrates in Syria), the bishop expresses his admiration for the Indians because of their valuable method of computation "done by means of nine signs." Severus had probably learned about the system from Eastern merchants active in Syria. This ingenious and eminently simple system of representing any quantity by using nine symbols in decimal place value (there was originally no zero) arose in India perhaps as early as the fifth century. The Indian system seems to have been known in Baghdad as early as 770, or less than a decade after its founding, but it was principally diffused through the writings of the Abbasid mathematician and geographer Muhammad ibn Musa al-Khwarizmi (or al-Khwarazmi), who died around 846.

Al-Khwarizmi, a native of the Khwarizm region of Central Asia, which is located just south and east of the Aral Sea, was attached to the caliph al-Mamun's House of Knowledge in Baghdad. Although al-Mamun's institute was instrumental in preserving Greek scientific and literary texts, paradoxically
none of al-Khwarizmi's Arabic mathematical works survive. The earliest known copy is a twelfth-century Latin translation, which opens with the words "duit Algoritmi" (Algorithmi says). In this manner, the mathematician's epithet of origin came in the West to denote the new process of reckoning with Hindu-Arabic numerals, called algorithmus in medieval Latin and algorism in English, and even the entire step-by-step process of solving mathematical problems, algorithm.

Al-Khwarizmi's efforts in ninth-century Baghdad should be understood in the broader context of the enormous interest in learning and science that was encouraged by the wider availability of paper. By the end of the century, Abbasid mathematicians had acquired and translated from the Greek all the writings that were to have a decisive effect on the growth of Arabic mathematics—including the works of Euclid, Archimedes, Apollonius of Perga, and Ptolemy—as well as the great Sindhind astronomical work in Sanskrit and writings in Middle Persian by pre-Islamic Iranian mathematicians. By using Indian numerals, Abbasid mathematicians were able to combine all these to produce a new arithmetic based on the consistent application of the previously known concept of decimal place value.

Although al-Khwarizmi's original writings on the subject are lost, scholars have reconstructed the workings of his Hindu-Arabic system from the writings of some of his successors and followers, such as Abu'l-Hasan Kushyar ibn Labban al-Jili, who composed his Principles of Hindu Reckoning around 1000. In it the author says that calculations are performed on a dustboard, called by the Persian name takht (table; throne). Calculation involved rubbing out and displacing numerals; the result of a calculation replaced one of the given numbers. For example, to multiply 123 by 456 the following figures successively replace one another on the dustboard:

\[
\begin{align*}
123 \\
456 \\
4 & 123 \text{ (multiply 100 by 400; insert 4 in the ten thousands' place)} \\
456 \\
45123 \text{ (multiply 100 by 50; insert 5 in the thousands' place)} \\
456 \\
45623 \text{ (multiply 100 by 6; erase 1 in the hundreds' place and replace with 6)} \\
456 \\
53623 \text{ (multiply 20 by 400; add 8000)} \\
456
\end{align*}
\]
54623 (multiply 20 by 50; add 1000)
456

54723 (multiply 20 by 6; erase 2 from the tens' place and add 120)
456

55923 (multiply 3 by 400; add 1200)
456

56073 (multiply 3 by 50; add 150)
456

56088 (multiply 3 by 6; erase 3 from the units' place; add 18)
456

As in earlier systems, the multiplications and additions are performed mentally, but in this case the interim results are entered and erased on the dustboard. This was still a tedious process, with plenty of room for error, because so much of the calculation was done mentally, making it difficult to check one's work. The handbook of arithmetic for government bureaucrats by the mathematician Abu'l-Wafa indicates that finger reckoning was still preferred for some operations. Nevertheless, Abu'l-Wafa sometimes employed Indian-type schemes and attempted to free them from the messy dustboard through, one may conjecture, the use of paper.

Already by the middle of the tenth century, several noted mathematicians had attempted to improve this cumbersome system by adapting it for use on paper. The Syrian mathematician Abu'l-Hasan Ahmad b. Ibrahim al-Uqlidisi ("the Euclidian"), writing at Damascus, applied Indian schemes of calculation to the old finger arithmetic as well as to sexagesimal (base 60) fractions. Al-Uqlidisi was also the first mathematician to alter the dustboard method to suit the use of ink and paper. Not only did he object to the awkwardness and messiness of dustboards, but he also frowned on their unsavory connotations, for most people still associated them with astrologers casting horoscopes. Significantly, al-Uqlidisi's method of calculation on paper showed separate steps, so revision was now possible. Although al-Khwarizmi's name is better remembered today, al-Uqlidisi's system of arithmetic is essentially the same as the one some of us were taught when young. We multiply each integer by each other integer, and then add up the results.

\[
\begin{array}{c}
123 \\
\times 456 \\
\hline
18 \\
120
\end{array}
\]
600
150
1000
5000
1200
8000
+ 40000
56088

We can follow the same route half mentally:

123
\times 456

\underline{738}
6150
\underline{+ 49200}
56088

Like multiplication on the dustboard, multiplication with ink on paper is broken down into simple steps; unlike multiplication on the dustboard, the individual steps are permanently retained for verification.

In retrospect, the new approach to calculation using Hindu-Arabic numerals and pen and paper appears to be an easy leap of the imagination, but practically it was not. Although it was eminently superior to the old system, most people were conservative and continued to prefer mental arithmetic, finger reckoning, and dustboards. Al-Jahiz advised schoolmasters to teach finger reckoning instead of calculation by means of the "Indian" numerals, and al-Suli gave secretaries similar advice. Even the great mathematician Abu’l-Wafa preferred finger reckoning for most secretarial uses. Al-Nadim, the well-informed and generally reliable author of the Fihrist, the survey of Arabic literature, gives Indian numerals perfunctory treatment. He does not even list al-Khwarizmi’s fundamental book, Calculation of the Hindu Numerals, among the list of the author's important works.

Similarly, mathematicians, astronomers, and astrologers continued in their writings to use words and the old abjad system of referring to numbers by the letters of the alphabet. Their writings, however, provide little evidence of how most mathematicians actually did their calculations, because the books are fair copies of finished works. It is likely that many mathematicians still used dustboards, despite their disadvantages, because paper remained too expensive to scribble on and throw away.

Nevertheless, some mathematicians may have used the new paper-based
system for purely mathematical work. Ibn al-Banna, for example, writing in the early fourteenth century, does not mention dustboards at all, which suggests that he did his calculations on paper, but most mathematicians were not "pure" mathematicians—they often earned their bread practicing the traditional sciences of astronomy and astrology, which remained staunchly attached to the older systems. The Ilkhaniid mathematician and astronomer Nasir al-Din Tusi wrote an entire work on dustboard arithmetic at a time—the thirteenth century—when good paper was readily available for books. The erasable dustboard, like the hornbooks and slate tablets of later times, continued to have the advantage of being cheap. Until the invention of the erasable pencil in the nineteenth century, writing was difficult to erase from paper. Indeed, one of paper's distinct advantages was that it provided a permanent record of the reckoning.

Ordinary people, too, seem to have kept to the old ways of calculation. Medieval Jewish traders, whose transaction records are preserved in the Geniza documents, continued to represent their numbers with words as well as with Hebrew and Coptic numerals, which are similar to the Arabic abjad system. Because these alphabetic systems do not lend themselves to arithmetical operations, merchants must have performed their calculations in some other way, but there are no traces in the Geniza materials of how merchants actually did their arithmetic. They could have used mental computation, but finger reckoning, dustboards, or even pen and paper may also have been used, for there would have been no reason to save in the Geniza storehouse a scrap of paper inscribed only with numerals. At first merchants may have been reluctant to use Hindu-Arabic numerals, but in the late fourteenth century, the historian Ibn Khaldun noted that practical business arithmetic using them was being taught in the schools of Islamic Spain.

Mercantile practice in the Islamic lands may have lagged far behind scientific theory, but despite scant initial interest in the new system, knowledge of Hindu-Arabic numerals spread fairly quickly. Hindu-Arabic numerals are first mentioned in the western Islamic lands about 950, when they were called "dust" numerals (huruf al-ghubar). Much nonsense has been written about the mysterious origins of dust numerals, but they are simply the type of Hindu-Arabic numerals that were used in Spain on a dustboard, the standard tool for computation at that time. Under the so-called Taifas, or Party Kings, successors to the Umayyad caliphate of Córdoba, which had collapsed in the early eleventh century, Muslim, Jewish, and Christian scholars met together at centers like Toledo to study scientific writings, particularly in astronomy and astrology, acquired from Eastern regions. The Hindu-Arabic system of calculation became unusually popular in Spain, thanks not to merchants who would
have recognized its inherent advantages but to mathematicians who owned or read copies of al-Khwarizmi's book *Calculation of the Hindu Numerals*. There, beginning in the twelfth century, it was repeatedly translated and adapted into Hebrew and Latin.

At the other end of Christendom, Byzantine mathematicians seem to have remained completely uninterested in the new system for several centuries. In the ninth century Abbasid mathematicians had been eager to acquire all the ancient mathematical knowledge they could from Byzantium, but the attitude was not reciprocal. Just as the Byzantines generally ignored the enormous opportunities afforded by paper, so Byzantine mathematicians failed to grasp the opportunity to learn about the new system directly from al-Khwarizmi's successors. The Byzantine emperor Michael III sent Photius, one of the most learned men of his time, on an embassy to Baghdad in 855, three years before Photius was named Patriarch of Constantinople, and Photius's interests, as well as his prolonged stay in Baghdad, enabled him to meet the major scientific personages in the city. Yet he apparently did not take knowledge of the new mathematics back home, for Hindu-Arabic numerals and the new method of reckoning with them remained unknown in Byzantium until 1252, when an anonymous work on Indian arithmetic was published. Maximus Planudes, who is generally credited with the introduction of Indian reckoning and the use of Indian numerals in Byzantium, is thought to have read this book.

In Spain, the transfer of mathematical knowledge from Muslims to Christians coincided with a sudden growth in the use of Hindu-Arabic numerals. European Christians, who since Roman times had done their calculations with movable counters on a type of abacus, began to inscribe the counters (apices) with the new numerals. The traditional Arabic numeral forms were rotated and transformed, first into the huruf al-ghubar, the dust numerals, and eventually into the "Arabic" numerals used universally in the West. A few numerals, such as 5 and 8, which is derived from a medieval scribal abbreviation of the Latin octo, replaced those commonly used by the Arabs (fig. 49). By the twelfth century, ghubar numerals had been absorbed into the Latin version of the algorithm (system of numeration), as seen, for example, in the work of the translator John of Seville.

The new technique of calculating with Hindu-Arabic numerals therefore spread throughout the Muslim world at exactly the same time as the new medium of paper. Some medieval mathematicians were quick to realize that paper and ink provided a better way to calculate with the numerals than the dustboard did, because the method left a permanent record of the calculation and allowed results to be checked, not just immediately after the calculation but as long as the paper remained legible. It also had the further advantage of using relatively little space. But paper, for all its advantages, was still just too
expensive to use and throw away. The spread of these numerals, therefore, was not the result of practitioners writing them on paper, with one user of the new system teaching another. Nor did it depend on practitioners using Hindu-Arabic numerals on dustboards. Rather, as in other contemporary fields of knowledge, books written on paper provided the medium by which knowledge was disseminated throughout the Muslim world, in this case reaching Christian Europe through the crucible of Spain and eventually transforming the ways Europeans used numbers. European Christians were themselves at first as reluctant as their Muslim peers to adopt the new system, and a struggle ensued between the abacists, who preferred traditional calculation with counters, and the algorists, who preferred manipulating the new numerals on paper.

Italian merchants were the first to consistently use Arabic numerals for business. Although commercial contact between Spain and Italy could have brought knowledge of the new Hindu-Arabic numerals to Italian notaries by the late twelfth century, Spanish merchants did not yet use the system, and it was introduced to central Italy through North Africa. The merchant Leonardo Fibonacci of Pisa (Leonardo Pisano), whose father had had commercial contacts with the Arab world, lived in Tunis and became familiar with Arabic numerals there. In his Liber abaci (Book of Apices), despite its name, he rejected the use of the abacus in favor of the Hindu-Arabic method of reckoning. As a result of his treatise, written in 1202, the use of Hindu-Arabic numeration caught on quickly in the merchant communes of central Italy and supplanted the use of the abacus. By the fourteenth century, pen and paper had by and large replaced the abacus as a calculating tool in Italian banks, and in yet another odd twist of history, European merchants trading in West Asia and North Africa appear to have been responsible for introducing their Muslim partners to the benefits of the system that Muslim mathematicians—but few Muslim merchants—had known and used for centuries.

**Commerce**

Although Muslim merchants did not immediately adopt Hindu-Arabic numerals, paper nevertheless played an important role in the development of
trade in the medieval Muslim world. Although we tend to think of the credit economy as a strictly modern development, medieval Muslim merchants conducted most economic activities on credit and recorded them on paper documents. The range of these documents—including but not limited to contracts, accounts, and letters of credit—shows that writing and written documents, particularly those on paper, were common at a time when the ideas of writing and written records were just beginning to enter the lives of Europeans and supplant the traditional reliance on memory and oral testimony. This is not to say, however, that commercial documents were written only on paper, for many ancient Babylonian tablets and Egyptian papyri record commercial transactions. Still, the quantity of commercial documents from medieval West Asia, especially those preserved for centuries in the Cairo Geniza, have allowed scholars to reconstruct much of the economic, social, family, and daily life of the time.

The demands of the state bureaucracy established by the Abbasids encouraged the use of paper for keeping government registers and accounts. Medieval historians report on the complexity of the recordkeeping. One tenth-century Khurasan tax collector kept accounts showing "the amount of assessed taxes, the amount paid by each taxpayer on account of the tax assessed, the journal containing daily income and expenditure, and the amounts totaled up at the end of every month. The yearly account was a register in which amounts paid in were systematically entered for easy reference. The statements were shown in three columns: first, the amount taxed; second, the amount actually collected; and third, the difference between the two. In most cases the amount paid in was less than the amount assessed. The quittance receipt for the tax. Final settlement. Release." Few, if any, of these government accounts have survived, so our knowledge of the workings of the medieval Muslim economy is often based more on interpretation of commercial papers, largely those preserved in the Cairo Geniza.

Trade in agricultural goods and the products manufactured from them, especially textiles, was the fuel that powered the medieval Muslim economy. Unification of the lands between the Atlantic and the Indian Oceans had created a vast common market the likes of which the world had never seen. Although the Koran and Islamic legal literature denounced all forms of lending money at interest, Muslim merchants had several other ways to acquire investment capital; and members of religious minorities were not prohibited from borrowing and lending with people outside their faith—the Geniza documents provide ample evidence of extensive trade and commercial relations between Jews and Muslims.

Trade between the furthest reaches of the Islamic empire was facilitated by the use of a single language (Arabic) and a single monetary system: the central
government was, at least in theory, responsible for minting gold dinars, silver dirhams, and copper fals at fixed weights and standards of fineness. This was, of course, the ideal. Although government mints struck coins to a weight standard in principle, coins were actually weighed and their value determined at every transaction, because the weight and fineness of individual coins varied from mint to mint and with the length of time they had been in circulation; coins became thinner and less valuable through wear. Weighing and assaying coins were tedious tasks, so—as earlier in Roman times and subsequently in Christian Europe—money was often handled in sealed purses, the exact value of which was indicated on the outside of the purse. Thus when a commercial document speaks of "a purse from the Treasury" whose value was seventy-six and one-sixth dinars, it undoubtedly contained eighty gold coins of less than full weight.

The profession of money changing was related to weighing and assaying, for an enormous variety of coins circulated at any one time. The documents from the Cairo Geniza tell us of coins minted in Iraq, Egypt, Tunisia, Morocco, and Spain, as well as in Norman Sicily and on the Italian mainland. These all had to be converted into the local currency to be legal tender. Oddly enough, there was no abstract term for money, at least in the period covered by the documents, the mid-tenth to the mid-thirteenth centuries.

The large number of coins in circulation meant that the inhabitants of the medieval Islamic domains were probably more accustomed to using money as a medium of exchange than were their contemporaries elsewhere, certainly in Europe. This is revealed by the size, variety, weight, fineness, and design of medieval Islamic coins. Nevertheless, most wholesale and even retail commerce was mainly conducted through the use of credit, recorded on sheets of paper. Immediate cash payments were rare and were usually rewarded by a standard discount of 2 to 4 percent. According to the eleventh-century legal scholar al-Sarakhsi, "Selling on credit is an absolute feature of trade. . . In most cases, profit can be achieved only by selling for credit and not selling for cash."

A large number of the Geniza documents deal with credit—for example, in acknowledgment of a debt (explaining when and how a debt will be repaid)—or with settlements and special arrangements connected with previous obligations. Other types of documents, including wills, inventories, accounts, and letters, also deal with credit and mention that collateral was provided by real estate, jewelry, clothing, bedding, and even books. Although Muslim, Jewish, and Christian law all prohibited the charging of interest, the documents show that borrowers consistently paid back more than they had borrowed. Sale on credit was normal business procedure, and loans were often disguised as deferred payments. The borrowing of relatively small sums was relatively common; merchants—both Muslim and Jewish—regularly lent each other money.
Commercial "paper," in use from at least the eighth century, made possible the transfer of large sums of money over considerable distances without the use of specie. Paper, as bureaucrats had discovered, had the inherent advantage of being more difficult to erase than papyrus or parchment, an extra protection against dishonesty in financial transactions. This system could—and did—lead to some complicated transactions, which were dealt with by means of such credit instruments as the hawala, a payment of debt through endorsement or transfer of a claim. (The Arabic word lies behind the French word *avale,* "endorsement.") An Alexandrian merchant wrote that "the purchasers of the pearls transferred payment to those to whom they had sold aromatic balls." A statement of release from the summer of 1052 tells of a North African merchant who purchased forty dinars’ worth of Egyptian flax on credit. He sold it in Tyre and Lattakya (both now in Lebanon), promising to pay it in Fustat (Egypt) by transferring his debt to one Nahray ben Nissim. This Nahray paid twenty-seven dinars, and the banker Abraham, "Son of the Scholar," paid the remaining thirteen dinars. One must assume that they both owed the North African merchant money or expected they could collect from him.

Another instrument of credit was the suftaja, a letter of credit or, technically, a loan of money to avoid the risks of transportation. In the vast stretches of the empire, there were bankers and banking but no banks, for there were no specialized institutions exclusively dealing in money. Well-known bankers usually issued letters of credit for a fee, and the suftaja could be drawn on like a modern money order, although they normally could not be transferred to another party. Around 1100 an Alexandrian merchant wrote to his correspondent in Cairo that he had sent the amount owed in cash because he had been unable to find someone willing to issue a suftaja. Suftajas could be sent over long distances, but only where direct and permanent business connections already existed. To illustrate: Geniza letters from Tunisia show that people anxious to send money to Baghdad had to send purses of coin to Fustat, where the suftaja might be issued.

Merchants also sent notes written on paper, known by the Persian term *sakk* or *sakka,* from which the European principle (and name) of a check is derived. At Awdagush, in the western Sudan, the tenth-century geographer Ibn Hawqal saw an officially certified check for forty-two thousand dinars drawn by a man from Sijilmasa on one Muhammad ibn Ali Sadun in the same city. The piece of paper had traveled across much of the Sahara. Around the year 900, an important man paid a poet with a check, but when he presented it, the banker refused payment. The disappointed poet composed a verse to the effect that he would gladly pay a debt of a million dinars on the same plan!

Credit permeated all levels of society. According to the Geniza documents,
one did not even pay the local grocer cash for daily supplies but sent written orders and settled after a certain figure—5, 10, or 20 dinars—had been reached. Even orders drawn on most bankers were relatively small, if we can judge from a collection of twenty orders drawn on a Fustat banker in 1140. Most of them ranged from one and a quarter to seven dinars; only one was for the relatively large sum of one hundred dinars. Orders of payment avoided cash transfers, and debts were endorsed over to other merchants, a religious foundation, or even a government office, although normally they were paid by a banker.

Cash discounts were granted when the payment was made promptly, a process called "speeding up" in the few Geniza documents that mention them. These discounts hovered around 3 percent, and were never granted on commodities that sold well. One Alexandrian merchant wrote his friend in Fustat, "Do not sell your caraway for less than three and one-half dinars per qintar, cash or credit; otherwise, keep it."

Although the suftaja has received scholarly attention, the paper economy reflected in the Geniza documents depended less on this device, whose role was rather limited, than on the pervasive ruq'a, which was both an order of payment or delivery given to a grocer, merchant or banker and a promissory note. A banker issued promissory notes only for people who had already made deposits, and the notes were issued in sums ranging from hundreds or thousands of dinars to fractions of a dinar. Promissory notes from established bankers were considered as good as cash. In one mid-eleventh-century case, a banker redeemed a suftaja for 44 dinars with promissory notes worth 20½, 18, and 4½ dinars; he paid the remaining sum of less than 2 dinars in coin. A century later, a man owed the administrator of a pious foundation twenty dinars, of which he paid seventeen dinars in bankers' notes and the rest in gold.

Although orders of payment dominated medieval economic life, actual paper money was tried only once—and disastrously. Paper money was used in China in a limited way as early as the ninth century. At first this "flying money" was more like a banker's draft, but by the eleventh century, the government had authorized sixteen private houses to issue notes and had even established an official agency to issue government notes, backed by cash reserves, in various denominations. Paper money circulated widely by the end of the Northern Song (1127) period, and Marco Polo described it with amazement. As a result of increased contact between China and Iran following the Mongol invasions of both countries in the thirteenth century, the Ilkhanid (Mongol) sultan of Iran Gaykhatu tried to introduce block-printed paper money in 1294. Gaykhatu's chao notes were introduced to the city of Tabriz, whose local population, even when threatened with death for their refusal, rejected them outright. The minister responsible for the disaster was
eventually executed, and the Ilkhanids returned to minting coins of the traditional Islamic type.

That we have any knowledge of economic activity in the medieval Islamic lands is largely because of the survival of written documents. In addition to commercial paper, which permeated economic life in the medieval Muslim world, paper was also used to keep accounts. Some Geniza merchants balked at providing written accounts and viewed requests as expressions of their partners’ mistrust, but in reality merchants regularly rendered exact accounts, often down to minute fractions of a dinar. Several types of accounts survive among the Geniza documents. They include reports to partners or customers about shipments, sales, purchases, and outstanding balances; accounts for specific connections or transactions, such as partnerships, sales, purchases, or transport; and accounts made for the writer’s own use. Another type of account was that submitted by community officials, which, oddly enough, lack a reckoning of total revenues and expenditures.

Short accounts were often written on single sheets, like letters, and were sometimes made in several copies, either for safety or because several partners were involved in the transactions. Multiple copies were also required in special situations, such as statements of indemnities to be paid after a ship foundered. Longer accounts were normally written in a narrow booklet format known as a daftar, which typically measured between 3–4 inches (7–10 centimeters) wide and either 5½–6 or 7½ to 11 inches (14–15 centimeters or 19–28 centimeters) tall. The format allowed the merchant to carry a daftar in his sleeve, which was normally quite wide and served as a pocket, to be readily available for reference. The daftar consisted of a single sheet folded into a bifolio, creating four sides for writing. It could be punched with holes near the fold, so that it could be assembled, presumably with cords, somewhat like a modern loose-leaf binder, to continue a long record. Apart from those from Tunisia, where parchment remained popular longer than elsewhere, virtually all Geniza daftars were written on paper. Because paper itself remained expensive, the typical order of payment was half the size of a modern check.

The Geniza documents hardly provide a complete picture of the financial state of the complex businesses these merchants ran. Ledgers as such do not survive among them, for instance. Account books were files into which a merchant entered notes about his payments and receipts, as well as copies of any accounts submitted to a business correspondent. The copy was called the "root" or "original" because it could be admitted in court as circumstantial, if not formal, proof. Merchants, like Muslim legal scholars, had an ambivalent attitude toward written documents, exemplified in a court record that reports, presumably with approval, that "one of the witnesses remembers the fact,
although it is not recorded in a daftar." Even though written accounts were essential to conducting business, skill and good memory remained equally important, even to merchants with complex commercial undertakings.

There is a close connection between the banking practices reflected in the Geniza documents of the eleventh century and those known from Europe two or three hundred years later. Accounting in the Geniza period followed well-established practices indicative of a long tradition and was a vital instrument in the maintenance of an orderly economy, although it did not attain the standards reached by the Italians in the late Middle Ages. The double-entry method of accounting used in late medieval Europe was, it has been suggested, of Arab or Indian origin. The suggestion, however, lacks supporting evidence, and it is safest to locate its origin in Italy toward the end of the thirteenth century, at a time when European merchants combined access to paper with calculation using Hindu–Arabic numerals; double-entry bookkeeping was, in fact, known throughout Europe as the "Italian" method. Later developments notwithstanding, financial procedure in the medieval Islamic lands was complex and sophisticated, and the whole complicated edifice depended on the availability of paper. It was used for communication over long distances, for recordkeeping, for transfer of cash, and for legal safeguards.

**CARTOGRAPHY**

Paper also played an important role in the development of medieval Islamic cartography, which inherited and expanded upon earlier traditions of Greek, Persian, and Indian cartography to create the most accurate representations of the world—and the heavens—before the development of modern cartography during the European Renaissance. The political and administrative requirements of Islam gave the ancient cartographic traditions a new impetus, but the flowering of Islamic mapmaking—like that of mathematics, new governmental administrative procedures, and complex intercontinental commercial enterprises—coincided with the expansion of a paper-centered culture in the ninth century. Greater interest in geography immediately resulted in a large number of geographical writings describing the world and its provinces and climates, although the earliest actual maps to survive date from after 1000.

Just as the explosion of writing and books in the Islamic Middle Ages should not hide the persistence of oral modes of expression among large segments of the population, the existence of maps does not signal the emergence of a map-reading public. Maps were neither available to nor meant for use by the masses; few people would have known how to read a map or even known that they needed one. Many medieval Islamic maps appear today to be particularly schematic and difficult to interpret (fig. 50). Nevertheless, the increas-
ingly sophisticated techniques of representing the three-dimensional world on a two-dimensional surface should be understood in terms of two trends: a growing awareness of the possibilities offered by large sheets of paper and a raised level of visual sophistication. I will discuss the latter in the next chapter.

For Muslims, knowledge of geography was of central religious importance. One of the fundamental pillars of Islam is to worship God five times daily, facing the Kaaba in Mecca, a direction known in Arabic as the qibla. Thus Muslims have to be able to determine from any point on earth the sacred direction of Mecca in order properly to pray and perform certain other ritual acts, such as burying the dead, reciting the Koran, giving the call to prayer, and ritually slaughtering animals. Mosques and other buildings for religious purposes were oriented toward Mecca; profane structures, such as bathhouses or latrines, in other directions. The Islamic idea of a sacred direction is comparable to the Jewish custom of praying toward Jerusalem, from which Muslim practice derives, and to the Christian notion of orienting churches to the east. The idea of a sacred direction, however, plays a much more important role in Islam, for it ultimately determines the orientation of much secular as well as all sacred space. Correct orientation, moreover, is obligatory on all Muslims, not optional, as it is to Christians.

Other aspects of Islamic ritual practice, such as the use of a lunar calendar
and the need to determine the correct time for the five daily prayers, promoted popular interest in observing the heavens. At a more scientific level, astronomers expanded classical traditions of mapping the stars and constellations and making astrolabes, the instrument used to calculate the position of heavenly bodies and ultimately one’s position on earth. The earliest Islamic map of the heavens is the painted domed ceiling of the early eighth-century bathhouse at Qusayr Amra, in Jordan. The decorative scheme was derived not from a planar projection on parchment or papyrus but from a celestial globe. The oldest surviving astronomical manuscript having illustrations of the constellations is Abd al-Rahman al-Sufi’s *Treatise on the Fixed Stars*, dated 1009–10 (fig. 51). The pictures show the configurations of the stars in the forty-eight constellations recognized by Ptolemy, but the figures are dressed in Oriental rather than classical Greek garb. Al-Sufi wrote in his text that although he knew of another illustrated astronomical treatise, he copied his illustrations directly from images engraved on a celestial globe, indicating that he was not working in a manuscript tradition. According to the eleventh-century scholar al-Biruni, al-Sufi explained that he had laid a very thin piece of paper over a celestial globe and fitted it carefully over the surface of the sphere. He then

**Fig. 51.** The constellation Andromeda from al-Sufi’s *Treatise on the Fixed Stars*, 1009–10. Ink and color on paper. Bodleian Library, Oxford (Ms. Marsh 144, p. 165)
traced the outlines of the constellations and the locations of individual stars on the paper. Al-Biruni later commented that this procedure "is an [adequate] approximation when the figures are small but it is far [from adequate] if they are large." The Oxford manuscript of al-Sufi's text was copied from the author's original by his son.

In the early years of Islam, when the community was small and restricted to Arabia, the relative direction of the Kaaba in Mecca was easy to determine. Any group who had traveled northeast from Mecca to Medina would know that the qibla was southwest, opposite from the direction in which they had traveled. As the Islamic world expanded to the shores of the Atlantic and the steppes of Central Asia, however, it became a greater challenge to determine the direction in which to pray. Medieval Muslims used two different approaches to determine the relative direction of Mecca: folk science and the mathematical traditions of classical and Persian scientific geography, which, unlike folk science, involved theory and computation.

Folk science divided the world into sections, which radiated from the Kaaba, and the qibla was determined by principles of folk astronomy. Thus, someone in the Maghrib prayed toward the "Maghribi" section of the Kaaba, which was defined by the rising or setting of some celestial body. By placing oneself in the opposite direction of the celestial body, one therefore faced toward the Kaaba. This easy approach was widely popular and advocated by most scholars of religious law. It is attested from the tenth century in treatises on folk astronomy and mathematical astronomy, almanacs, geographies, cosmographies, encyclopedias, histories, and legal works but was probably used much earlier.

Like scientific astronomy, scientific geography was the speciality of a select few. It was based on the application of mathematical principles of geometry and trigonometry to the determination of longitudes and latitudes, as well as on knowledge of geodesy (the measurement of distances on the curved surface of earth). Islamic geography incorporated several different geographical traditions, the most important being the classical tradition of the second-century Alexandrian geographer Ptolemy and his successors, who gave localities coordinates of longitude and latitude and divided the earth into zones, or climes, according to the length of the longest day in the center of the zone. Many mathematicians were naturally interested in the problems of geography.

From the very few and ambiguous references to geographical representations in early Islamic times, we can guess that maps were neither widely known or widely used. Around 702, for example, a "picture" of Daylam, the mountainous province to the southwest of the Caspian Sea, was prepared for al-Hajjaj ibn Yusuf, the Umayyad governor of the eastern part of the empire, so that
he could better understand the military situation there. The same governor is also said to have ordered a picture of Bukhara so that he could prepare for its siege in 707. A picture of the swamps of al-Batiha, near Basra, was available during the reign of the Abbasid caliph al-Mansur to settle a dispute about water rights. We have absolutely no idea whether these pictures were actual maps, schematic plans, bird’s-eye views, or something else, but they were rare enough for later historians to single them out for mention.

A consistent and coherent system of mapmaking did not evolve until the early ninth century, during the reign of the Abbasid caliph al-Mamun, when the House of Wisdom in Baghdad was in full operation. Scholars there translated and even improved on earlier Greek, Persian, and Indian geographical works, particularly Ptolemy’s Geography. Al-Mamun himself is known to have commissioned a large colored representation of the world, known as the mamuniyya. “representation.” A century or so after the events in question, the historian and geographer al-Masudi reported that the caliph had ordered a group of scholars to represent the world with its spheres, stars, lands, and seas, the populated and unpopulated areas, settlements, cities, and the rest. Al-Masudi said that the representation was better than anything that had preceded it, whether the map in the Geography of Ptolemy, his successor Marinus’s map, or any other.

No trace of al-Mamun’s map remains, and its actual form is uncertain, although it must have been quite large. On the one hand, al-Masudi’s comparison with Greek maps suggests that it, like them, was built up from longitude and latitude tables on a gridlike projection. On the other hand, references to later, but equally lost, copies of al-Mamun’s map suggest that it may have been based on the Persian geographical system of seven climes (kishwar), in which a central capital region is surrounded by six provinces. Topographical accuracy does not seem to have been of prime concern, for al-Mamun, like other rulers before and after him, probably ordered his “picture” to show that he, placed in the center, ruled over everything that mattered, which was spread around him.

Although al-Mamun’s world map may have been designed primarily to massage the caliph’s ego, it is still likely that the mathematician al-Khwarizmi was one of the group of contemporary scholars whom the caliph had chosen to make it. Al-Khwarizmi’s fame in the West was established by his books on Indian numerals and algebra, but in the medieval Islamic world his astronomical and geographical works were at least as significant. Al-Khwarizmi wrote a book containing tables that located 2,404 localities by exact longitude and latitude. These corrected many erroneous values given by Ptolemy and gave the coordinates of many more locations.
Because Ptolemy's coordinates were given to mark places on a map, it seems likely that al-Mamun's map would have used a similar system of location by means of coordinates. The likelihood is strengthened by another connection: Al-Khwarizmi's tables provided a model for those of the geographer Suhrab ibn Sarabiyun, who did give detailed directions for producing a map on a rectangular grid from lists of coordinates. First, draw a rectangle, the larger the better, and then divide its edges into degrees, mark the equator, and insert the horizontal lines dividing the climes. To pinpoint features on the map, stretch a thread due north and south at the required longitude, and stretch another thread due east and west at the required latitude. Mark the point of intersection.

Suhrab's text suggests that al-Mamun's earlier map followed Greek precedent in yet another important way, for he indicates that east should be to the right and west to the left on the map. North would therefore be at the top of the map, following the Greek principles of orientation. By the time Suhrab's manuscript was copied in Arabic in the tenth century, however, normal Islamic practice put south at the top of maps, probably because Mecca was to the south of most localities represented on their maps. Although putting south at the top is also characteristic of Chinese maps, the Islamic development appears to have been independent.

It is thought that al-Khwarizmi derived the coordinates for his tables not by copying earlier tables but by placing a grid, like the one that Suhrab describes, over a Syriac version of a Ptolemaic map. This idea is supported by al-Khwarizmi's confusions and misreadings of certain place-names and coordinate values, which can best be explained by the difficulties of translating names and numbers from Greek and Syriac into Arabic. In all these languages, as we have seen, numbers were normally represented by alphabetic signs, and in the Semitic alphabets, the frequent omission of diacritical marks to distinguish letters of similar shape but different numeric value would have caused mixups and garbling of unfamiliar names and values. Sometimes, for example, geographical coordinates rendered in abjad could be read as unfamiliar place-names!

Texts do not specify the material from which al-Mamun's map was made. Because it was large, it may have been drawn on cloth, like other large maps from later times, for paper was not yet made in large sheets. In the tenth century, al-Nadim, the author of the Fihrist, saw a "description of the world" prepared by a Sabian from the city of Harran, in northern Mesopotamia. He noted that it was on linen, "unbleached, but with colors," which suggests that this description of the world was actually a map on cloth. At roughly the same time, in 964, the Fatimid caliph al-Muizz, who then ruled in Tunisia, ordered a "picture" woven of blue tustari silk, portraying the earth's climes, mountains,
seas, cities, rivers, and roads. Representations of the two holy cities, Mecca and Medina, were prominent. Every detail was identified in writing, presumably embroidered, in gold, silver, or colored silk threads. The picture is reported to have cost the stupendous sum of twenty-two thousand dinars to prepare and seems to have been intended to fulfill much the same purpose as al-Mamun’s representation: to exalt the caliph who commissioned it.

The earliest extant maps from the Islamic world, however, are on paper. They are included in a unique manuscript of al-Khwarizmi’s geographical work discovered in Cairo at the end of the nineteenth century. This manuscript, dated 1037, contains four sketch maps showing the Island of the Jewel, the World Ocean, the Nile, and the Sea of Azov (fig. 52). Because the maps were made almost two centuries after al-Khwarizmi wrote the text, it is unclear to what extent they reflect al-Khwarizmi’s original concept and to what extent they reflect a convergence with a second tradition of Islamic cartography, associated with the florescence of geographical writing in the tenth century.

Within little more than a century a new genre of Islamic literature had developed. Its practitioners were not merely geographers: their works contained much historical, economic, and sociological information. Maps were
necessarily an integral part of this genre. Apart from the writings of Ibn Khur-
radadhbih, whose geographical manual was designed for use by secretaries in
government administration, most of the geographical works produced in the
tenth century belong to the genre of "routes and kingdoms" (al-masalik wa'l-
mamalik). Their authors were largely travelers who had gathered firsthand docu-
mentation about the geography of the Islamic world. These include al-
Yaqubi, who wrote the Kitab al-buldan (Book of Lands), al-Balkhi, who wrote the
Kitab suwar al-ard (Book of the Depiction of the Earth), al-Istakhri, who wrote
the Kitab al-masalik wa'l-mamalik (Book of Routes and Kingdoms), Ibn Hawqal,
who wrote the Kitab surat al-ard (Book of the Picture of the Earth), al-Muqadd-
dasi, who wrote the Kitab ahsan al-taqasim (Book of the Finest of Selections), and
al-Bakri, who wrote the Kitab al-masalik wa'l-mamalik (Book of the Routes and
Kingdoms). Rather than divide the world into climes, these authors distin-
guished large regions roughly corresponding to contemporary political enti-
ties (mamalik), about which they recorded general features, including the cli-
mate and people, and assessed how the details of each place bore on present life
there. The relationships among the different authors, the many manuscripts,
and the surviving sets of maps in these geographical works are extremely dif-
cult for scholars to disentangle. Because al-Muqaddasi wrote that al-Balkhi's
book (which has not survived) represented the earth with very carefully pre-
pared maps, this tradition of mapmaking is commonly known as the Balkhi
school. The number of surviving examples suggest that it was very popular in
the Middle Ages.

Many, if not all, of these geographical works were—or were meant to be—
accompanied by regional maps of the Islamic empire, and a complete set of
maps contained between twenty and twenty-two separate representations.
Indeed, the texts sometimes specify that they are meant to be read as accompa-
niments to the images, suggesting that authors were thinking in new, more
graphic ways. We know that al-Mamun's map was colored, but we have no indi-
cation that the different colors used were linked to specific geographical fea-
tures. A century and a half later, however, al-Muqaddasi explained that colors
are significant on his map: "We have colored the familiar routes red, the
golden sands yellow, the salt seas green, the well-known rivers blue, and the
principal mountains dull brown." Al-Muqaddasi's text is particularly con-
cerned with the relative importance of the features described. On his maps he
indicates the relative importance of towns, for example, by varying the size of
the circles that represent them.

A complete map set in the Balkhi school normally consisted of a world
map, maps of the three seas—the Mediterranean, the Persian (Gulf), and the
Caspian—and seventeen maps of "provinces," or regions. In complete contrast
to maps based on the Ptolemaic system, the Balkhi set had no mathematical basis in latitude and longitude. The Balkhi world map should be understood, therefore, as an armchair attempt to see all the provinces set down relative to each other and fit them into a stereotyped idea of the world. The inhabited hemisphere is represented by a circle in which the Mediterranean and Persian Seas are complementary shapes.

This schema is somewhat comparable to, but quite different from, the tripartite "T-in-O" mappae mundi, or world maps, of the Western medieval tradition. In these, the O represents the ocean surrounding the disc of the earth. Within the circle, the vertical stroke of the T represents the Mediterranean Sea, which divides Europe on the right from Africa on the left. The horizontal stroke of the T is formed by the River Don on the left and the Nile on the right, which separate Asia (at the top) from Europe and Africa on the bottom left and right. This tripartite division, which conveniently places Jerusalem at the exact center of the map, also symbolizes the Trinity and the inheritance of Noah's three sons. The largest of these medieval mappae mundi to survive is a thirteenth-century example on parchment in Hereford Cathedral.

The number (normally thirteen) and consistency of the Balkhi school maps that represent the Persian-speaking regions show a distinct bias toward the Iranian lands, suggesting that the set derives from an earlier, perhaps pre-Islamic Iranian tradition of representation. In contrast, the maps of Egypt and the Maghrib—both Mediterranean regions—vary enormously from manuscript to manuscript, suggesting that the Balkhi geographers were quite unfamiliar with the Ptolemaic cartographic tradition. In no case can the individual maps be joined together, like the pieces of a jigsaw puzzle, into a larger whole.

Most Islamic maps followed the style of the Balkhi school, but the increasing availability of paper, particularly in the eastern Islamic lands, seems to have encouraged geographers and others to design their own maps. The Turkish grammarian al-Kashgari wrote a book on the Turkish language in 1076–77. The only surviving manuscript of his text is dated 1266 and contains an unusual map of the world, which may be a fair copy of the author's original design (fig. 53). Centered on the Turkish-speaking areas of Central Asia, with other countries receding in size as they approach the circumference, the map prefigures by some seven centuries Saul Steinberg's celebrated New Yorker cover, in which the lands west of the Hudson River recede into faint and provincial nothingness.

Around the year 1000, the polymath al-Biruni grappled with the most appropriate method of representing the spherical surface of a globe on a flat surface. He came to this problem in the context of his own scientific work in the plains of what is now Pakistan, where the slight curvature of the earth that he observed led him to the conclusion that the earth was round. Al-Biruni may
have also been brought to this question because he had tried to determine the correct length of one degree of latitude. In his book comparing the calendrical systems used by different peoples of the world, he devoted one chapter to several methods for projecting star maps, and a few years later he composed a monograph on the same subject, probably for Abu’l-Hasan Ali, the Khwarazmshah ruler of Central Asia who died in 1008–9.

Although al-Biruni mentioned terrestrial maps, his focus was on celestial maps, and he described seven methods of projecting the celestial sphere onto a flat surface. The first four of al-Biruni’s methods were derived from earlier sources, including Ptolemy’s Geography and several Abbasid scientific works, but the last three were quite original. The first of his original methods was a globular projection resulting in four maps; the second employed dividers to measure the distances between stars on a globe and transfer them to a map; and the third involved marking the stars on a globe with a substance that would transfer to a flat surface when the globe was rolled with a circular movement, producing in effect a sort of monotype. Whatever the practicality or impracticality of al-Biruni’s seven celestial projections, they indicate an increased interest in graphic modes of representation around the year 1000, encouraged, no doubt, by the wider availability of and increasing familiarity with paper.

The outstanding figure in Islamic cartography is the Sharif al-Idrisi. Born to a noble family in Ceuta in 1100, he traveled widely in Morocco and Spain and even ventured to southern France and the English coast. Around 1138, King Roger II of Sicily invited him to Palermo to construct a world map and write a commentary on it. The map and the accompanying book, Nuzhat al-mushfaq (Pleasant Journeys; also known as the Book of Roger), were completed in January 1154. The irony is that al-Idrisi, who had been invited to Roger’s court because of his impressive genealogy, which went back to the prophet Muhammad (hence his title Sharif), had little knowledge of cartography at first. Even-
tually, however, he came to be regarded as one of medieval Europe's foremost geographers and cartographers, for his work combined knowledge of the Ptolemaic and Balkhi traditions with a distinctly new and graphic conception of the world.

Al-Idrisi's method was refreshingly practical and original, probably because as a novice in the field, he was unencumbered by generations of cartographic and geographical scholarship. For about fifteen years, according to his own account, he studied extant geographical writings. Finding the writings contradictory, he discussed the subject with scholars, but they were not much more help, so he talked with well-traveled people, who told him what he needed to know about foreign places. He collated the pertinent material by entering it on a "drawing board," using "iron instruments" to inscribe items mentioned in books, together with the more authentic of the scholars' and travelers' information. He then prepared a pure silver disk weighing four hundred Roman rati (a measure somewhat equivalent to a pound), on which he engraved the design transferred from his drawing board. It showed the seven climes and their lands and regions, shorelines and hinterlands, as well as other details.

Neither al-Idrisi's drawing board nor his silver disk survives, but six later manuscripts of his Pleasant Journeys preserve a small circular world map, which appears to be a reduced version of the silver original. The earliest is a copy (on paper) dating to 1300 (fig. 54). Several features, such as the location of south at the top and the general shape of the continents, closely follow world maps of

Fig. 54. World map from a copy of al-Idrisi's Geography made in 1300. Ink and color on paper.
Bibliothèque Nationale de France, Paris [Ms. Arabe 2221, fols. 3b–4a]
the Balkhi school. New elements are the curved boundaries of the seven climes and the division of each clime into ten sections. Unlike the provincial maps of the Balkhi school, the sectional maps that survive in eight early al-Idrisi manuscripts can actually be fitted together into a grid, like maps in a road atlas (fig. 55). Because the cartographic style varies considerably between the different manuscripts, a consistent vocabulary of geographical representation seems not yet to have evolved, however.

Al-Idrisi’s world map was unusual in combining the Islamic circular world map, ultimately derived from the idealized maps of the Balkhi school, with the coordinate system, or graticule, known to the Greeks. Although the graticule may have been used in al-Mamun’s map, it was not widely used in the Islamic lands, even though geographers there did commonly use terrestrial and celestial coordinates to locate positions. Al-Idrisi’s adherence to the old system of the seven climes made his attempt somewhat unwieldy, but his set of maps shows an increased graphic sophistication, with each small map keyed exactly to its position on the larger world map.

Al-Idrisi’s original world map was engraved on silver to guarantee its preservation, but its production is inconceivable without the use of paper, for paper was perfectly suited not only to assembling all the relevant information but also to transferring it either from or to a rounded surface. In the end, the bullion value of the silver seems to have guaranteed just the opposite fate for the disk, and only paper copies of al-Idrisi’s text and maps survive. Similar concerns about the durability of paper led the Norman rulers of Sicily to recopy paper records onto parchment, with better results.

The final development in Islamic cartography before it was transformed by the European cartographic tradition occurred in early fourteenth-century
Iran, another crucial period for the history and development of paper in the Islamic lands. Just at the moment when many types of books were becoming larger and illustration was taking on a new and important role, the Ilkhanid geographer Hamdullah Mustawfi prepared two world maps for his geography, *Nuzhat al-qulub* (Diversion of the Hearts). Although no copies of Hamdullah Mustawfi’s maps survive from the author’s lifetime, a later copy of his map showing the eastern Islamic lands takes off in an entirely new direction. The map identifies the climates according to the earlier system, but it differs completely from earlier maps in showing no linear features except for coastlines. Instead, the map is ruled into a graticule of squares, each one degree of latitude wide and one degree of longitude long. Localities are placed within individual squares (fig. 56).

This new type of map did not derive from the Islamic cartographic tradition but from the Chinese, a product of increased contacts between Iran and China during the thirteenth and fourteenth centuries. The date and place at which this gridded map appeared suggests that Mustawfi must have seen and been inspired by Chinese maps, which often used grids. Mustawfi’s map differs, however, from Chinese grid maps, which are based on linear measurement on the ground and not, like Islamic maps, on the angular measurement of latitudes and longitudes. Still, Mustawfi’s map is similar to Chinese

![Graticule map of central and western Asia by Hamdullah Mustawfi, a 16th-century copy after a 14th-century original. British Library, London (Add. Ms. 16796, fols. 143b-144a) ](image)
prototypes in placing the represented spot in the relevant square instead of on a particular point.

Similar gridded maps appear in the work of the Timurid geographer Hafiz-i Abru, but by the end of the fifteenth century, Islamic geographical cartography was in decline in face of the development of a more sophisticated European tradition; Europeans soon had direct knowledge of places that no Muslim geographer or traveler had ever visited. In the Ottoman world, cartographers participated in the explosion of European cartography, but outside it, most maps made later in the Islamic lands continued to reproduce, with lesser degrees of accuracy, the great achievements of the past.

Medieval Islamic geographers could have achieved their cartographic triumphs without paper. Earlier Greek geographers had created maps of lasting importance without even knowing about paper. Nevertheless, the easy availability of paper in medieval Islamic times encouraged new graphic modes of thinking, and the coincidence of papermaking with mapmaking in the ninth and again in the thirteenth to fourteenth centuries can hardly have been fortuitous. The connection becomes all the more convincing when we look at other contemporary systems of notation developed for use on paper.

**Music, Genealogy, and Battle Plans**

In the medieval Islamic lands, as in the ancient world, music was closely related to mathematics. Not only were pitches and meters governed by mathematical principles, but, as in mathematics, the study and performance of music were largely anchored in memory and gesture—in other words, in a system in which one musician learned directly from another and played by ear. Although most West Asian musicians have transmitted their knowledge by memory and gesture to the present day, Arabic music underwent important developments as a substantial corpus of theory was written down and as musical theorists attempted to develop systems of notation. The first period of development occurred under the Abbasids in ninth- and tenth-century Iraq; the second, under the Ilkhanids in thirteenth- and fourteenth-century Iran. These two attempts to transcribe sounds and rhythms coincide with the two most important periods in the development of paper in the Islamic lands. It makes sense to think that individuals who already knew the possibilities of paper in one realm of knowledge were eager to realize its possibilities in others.

The ancient Egyptians, Hebrews, Chinese, and Greeks had all come up with very different systems to represent their music. Just as the ancient Greeks had used the built-in ordering of the alphabet to represent numbers, they represented pitch with a mixture of letters and signs. By the ninth century, many different systems were being used. The Byzantines used this ancient system of
ekphonetic notation, and Nestorian Christians may have transmitted their own version of it as far as Tibet, where Buddhist monks employed it to notate their chants. At the monastery of St. Gall, in Switzerland, monks used neumatic notation, with finely drawn lines, curves, and hooks representing the rise and fall of the melodic line in their plainchants. This was also the time when Arab musical theorists first developed an alphabetic system of notation for Arabic music. Because of the differences between musical theory and practice, their system was never actually used in the ninth century, and other attempts at notation were made in the thirteenth century.

The major figure in Arabic music of the early ninth century was the philosopher Abu Yusuf Yaqub ibn Ishaq al-Kindi. Son of the governor of Kufa, he was educated in Basra, a lively intellectual center, and became attached to the Abbasid court in Baghdad during the reigns of al-Mamun and al-Mutasim. There he studied Greek philosophy and wrote several short treatises on music, of which at least five have survived. Eclectic in approach, they show his dependence on the Aristotelian and Platonic-Pythagorean traditions. In his musical treatises he outlines melodic movement schematically in words, implying visual metaphors with terms like lawabi (spiral), dafir (braid), and muwashasha (girdled). His descriptive account of contemporary rhythmic cycles is somewhat puzzling and imprecise, possibly because it was a pioneering attempt quite independent of previous analytical models.

Al-Kindi is particularly interesting because he discussed a model of musical notation that several later theorists borrowed or even reinvented. He denoted pitches with letters of the alphabet, ordered in the abjad sequence, making his system alphabetic-numeric. It was, however, confined to purely theoretical discussions, for when al-Kindi defined the pitch outlines of an elementary lute exercise, he did not use notation to record them but preferred verbal definitions of the frets used. Al-Kindi's efforts to write about music derived in part from Greek traditions, but, like many of his contemporaries in other fields, he was also encouraged by the flowering of books and book learning associated with the spread of paper use from ninth-century Baghdad. Musical notation is scarcely conceivable without a writing medium, and paper was readily available.

Al-Kindi's efforts apparently went nowhere, but some of his ideas reappear in the late thirteenth century in the most influential of all later treatises on music, the Kitab al-adwar (Book of Cycles) by Safi al-Din al-Urmawi, a native of Urmiya, in northwestern Iran. The book gets its name from the author's representations of the basic set of common rhythmic cycles in the form of a circle. The perimeter of the circle is inscribed with the appropriate syllabic sequences, which are qualified by a verbal statement of the number and posi-
tion of those time units marked by percussion. The circles are also used to indicate the number of consonant relations in some of the modes, which are shown as lines across the circle joining the notes in question inscribed around the perimeter. Because Safi al-Din uses letters for pitch and numerals for duration, his system also gives some indication of rhythmic structure. It defines rhythmic cycles and their variant forms in terms derived from prosody to specify the various internal divisions and accessional patterns. Such a scheme seems impossible to imagine without some writing material, and paper was readily available in thirteenth-century Iran.

Safi al-Din did not intend to provide an accurate record of the musical repertoire, which would have served no useful purpose in a tradition in which aural transmission and lengthy and free variations on simple themes were the norm. Rather, he aimed to represent the melodic structure of particular forms and demonstrate that a technique of notation was possible. His are the first precise depictions of musical meters; letters and numbers note melodies. His book was the most popular and influential book on music for centuries, and no other Arabic (or Persian or Turkish) music treatise was so often copied, commented on, and translated.

Safi al-Din may have been prompted to develop his system by the peculiarities of his education, which deeply involved him in the paper-centered culture of his time. Born around 1216, he went to Baghdad in his youth, where he was educated in the Arabic language, literature, history, and penmanship. He studied Shafii theology and comparative law at the Mustansiriyya madrasa, eventually assuming a post in the caliph's juridical administration. He also made quite a name for himself as a calligrapher, working as a copyist at the new library built by the caliph al-Mustasim. His disciples included both Yaqut al-Mustasimi, the cynosure of calligraphers, and his famous student Shams al-Din Ahmad al-Suhrawardi. Safi al-Din also became known as a musician and excellent lutenist, talents he was able to exploit when he was accepted into the caliph's circle of boon companions. Safi al-Din wrote the Book of Cycles while he still worked in the library of al-Mustasim; the earliest known manuscript was finished in 1236, when the author was only twenty years old or so. Because the handwriting closely resembles Yaqut al-Mustasimi's, it may well be the author's own copy.

After the fall of the caliphate to the Mongols, the Ilkhanid governor of Iraq, Ala al-Din Ata-Malik Juwayni, and his brother Shams al-Din Muhammad Juwayni put Safi al-Din in charge of the chancellery of Baghdad. He was also appointed chief supervisor of charitable foundations in Iraq until 1267, when the noted astronomer Nasir al-Din Tusi took over the position. Safi al-Din's ample private fortune helped him survive the fall of Baghdad:
having graciously accommodated one of the Mongol officers, the officer intro-
duced him to the new ruler, who was impressed by Safi al-Din’s art and erudi-
tion and doubled his income. Safi al-Din’s later musical career was supported
mainly by the Juwayni family, but after the demise of his patrons in 1286, he
fell into oblivion and poverty.

Safi al-Din’s alphabetic and numeric representation of pitch and duration
was systematically expanded by his first great successor, the Persian Sufi poly-
math Qutb al-Din Shirazi. Qutb al-Din was born into a family of physicians
in 1236 but was not only a medical man; he also distinguished himself by writ-
ing on philosophy, astronomy, and religious problems. Sufis often flaunted
norms, and as a Sufi, he neglected some of his religious duties, drank wine,
and associated with people of ill repute. He played chess brilliantly and was also
skilled as a conjurer and musician. In a unique example of a fully notated piece
of music—a song by Safi al-Din—Qutb al-Din used a grid in which the divi-
isions along the horizontal axis represent time units, and superimposed layers
provide for pitch, the text, a percussion part, indications of expression and
dynamics, and specifications of changes of mode. His use of a grid came at vir-
tually the same moment that cartographers were beginning to use grids for
mapmaking, and, as we shall see in Chapter 5, builders were using grids for the
representation of architecture.

Qutb al-Din’s unique effort was revisited a century later by the Persian
musician Abd al-Qadir ibn Ghabi. His major treatise, the Jami al-alhan, gives a
succinct description of the major structural features of each type of song, not-
ing any associations with particular rhythmic cycles and, more characteristi-
cally, with verse in a particular language or form. He also provided an extended
analysis of one piece of music, including skeletal notation; the various sections
are segmented, and aspects of verse setting are illustrated.

Another form of notation that developed as a function of the increased use
of paper was the genealogical tree. The study of genealogy, known in Arabic as
nasab, was fundamental to Arab society because it validated kinship and all that
kinship involves. Genealogical research was practiced from very early—even in
pre-Islamic times—although none of the earliest genealogical works has sur-
vived. The study of genealogies, like other sciences, flourished under the
Abbasids in the ninth century, when writers such as al-Baladhuri wrote Ansab
al-ashraf, a multivolume work concerning the genealogy of the sharifs, or
descendants of the Prophet. Far more interesting for our purposes, however,
is that genealogical writers began at the same time to present the essence of
their research in graphic form—a form known either as tashhij or mushajjar, from
the Arabic word for "tree." Some of the earliest examples appear in al-Ham-
dani’s al-Ikhtil (The Crown), a tenth-century encyclopedia of south Arabian
archaeology and genealogy, known only (and incompletely) from later manuscripts. Once again, the introduction of paper and the development of new graphic modes of representation coincided.

The use of genealogical trees became more frequent over time; trees did, after all, provide an immediate understanding of relationships, as the fourteenth-century historian Ibn Khaldun averred. An example of a tree can be found in the early fourteenth-century copy of the Ilkhanid vizier Rashid al-Din’s Majmu’a al-rashidiyya, a collection of his theological writings, where a graphic genealogy beginning with Adam and ending with the family of Fatima, Muham-
mad's daughter, covers nearly twenty-five pages. Some of Rashid al-Din's historical works were illustrated with genealogical charts in which individuals were represented by more or less schematic portraits. The originals of these charts have not survived intact, for the portraits were later excised from the manuscripts and pasted into albums, but the charts are known from later copies.

Yet another form of graphic representation that developed in medieval Islamic manuscripts was the battle chart; such charts are sometimes found in manuals of furusiyya, or theoretical and practical equestrian knowledge. Ninth-century writers, such as al-Jahiz, explored the equestrian field, as they did other branches of science, but the full development of the equestrian manual did not occur until the thirteenth century, under the Mamluk rulers of Egypt and Syria. The later manuals described the skills necessary for a successful military career, including knowledge of horsemanship, archery, fencing, and games, ranging from polo to chess. Some of these manuscripts, such as the Nihayat al-su'ul (End of Questioning) written by al-Aqsarai, have schematic illustrations and even charts that represent the tripartite battle formation, with the crack troops in the center under the personal command of the "king"; his royal banner flies above them. He is flanked by right (maymana) and left (maysara) wings (fig. 57). Because no earlier manuals of this type are known to survive, it is impossible to say whether such battle charts were used earlier, but the complexity of the chart is well served by the generous two-page spread in the manuscript, making the chart a full 16 inches (40 centimeters) wide.

All told, the convergence of so many new and transforming forms of notation in the ninth and the thirteenth to fourteenth centuries cannot be mere coincidence, especially since so many of the individuals involved in one were involved in another. Men of letters familiar with paper would have been quick to recognize its advantages in various areas of study. Paper was thus the catalyst for many of the intellectual developments of the Islamic Middle Ages. It was also the catalyst for many simultaneous developments in the visual arts.